Why Is the Sovereign Greenium Small? Commitment, Pass-Through, and Additionality*

Diogo Sampaio Lima
Maastricht University

Last updated: June 30, 2026

Abstract

Sovereign green bonds usually promise ordinary sovereign cash flows together with use-of-proceeds commitments. This institutional fact makes the bond-level greenium a misleading sufficient statistic for the value of the commitment. If a credible green covenant improves fiscal capacity and lowers default risk, the benefit should be capitalized into all pari-passu claims on the issuer, not only into the labelled bond. I formalize this idea by decomposing the observed greenium into an issuer-wide commitment component, a bond-specific residual wedge, and pass-through to conventional debt and CDS. The empirical evidence is consistent with this decomposition: sovereign greeniums are close to zero in yields, modest in asset-swap and z-spread measures, issuer-risk event responses are small, and official allocation reports reveal substantial heterogeneity in credibility, adaptation content, and accounting additionality. I then develop a long-term sovereign default model with climate risk, adaptation capital, and green-obligation states. In the Brazil benchmark, green access lowers mean spreads, reduces annual default risk, and raises initial-state welfare by 1.29 percent relative to regular-only borrowing, even though the model-implied bond-level commitment gain is modest. Mandatory green issuance is welfare reducing. The results imply that sovereign green bonds are best understood as state-contingent commitment options rather than as universally cheaper debt.

JEL classification: F34, G12, H63, Q54.

Keywords: sovereign debt, green bonds, default risk, climate adaptation, commitment.

*Email address: d.sampaiolima@maastrichtuniversity.nl.

1 Introduction

Sovereign green bonds have become a standard instrument of public debt management. Governments use them to signal climate priorities, broaden the investor base, build sustainable-finance benchmarks, and finance expenditures classified as mitigation, adaptation, resilience, or transition investment. The institutional package is familiar: a framework, an eligible-expenditure list, external review, allocation reports, and impact reports. Yet the financial claim is usually not a project-finance claim. It is ordinary sovereign debt: pari passu with the issuer's conventional bonds and exposed to the same default event.

That institutional fact creates the central puzzle of the paper. If a green covenant improves the sovereign's repayment capacity, why should only the green bond become safer? Conversely, if measured greeniums are small, should we conclude that sovereign green bonds have no fundamental value? The answer matters for how governments and international institutions evaluate these instruments. A small measured greenium could mean that investors attach little value to the label. It could also mean that the value of the commitment is capitalized into all claims on the issuer, leaving only a small residual wedge between the labelled bond and its conventional comparator.

This paper argues that the answer to the second question is no. The bond-level sovereign greenium is not the right sufficient statistic for commitment value. A credible use-of-proceeds covenant can change future policy and lower default risk, but that effect is issuer-wide. Conventional bonds and CDS should also reflect the improved repayment state. The relative yield between a green bond and a matched conventional bond therefore captures only the residual part of value that remains bond-specific after issuer-wide repricing: label demand, liquidity, scarcity, index eligibility, clientele effects, taxes, benchmark status, and measurement error from imperfect counterfactual bonds. A small observed greenium can coexist with meaningful commitment value.

The paper formalizes this idea with the decomposition

$$\mathcal{G}^{obs}_{bct} = \mathcal{G}^{F}_{ct} + \Lambda_{bct} - \mathcal{P}_{bct} + \mathcal{R}_{bct}.$$

Here $\mathcal{G}^{obs}_{bct}$ is the observed bond-level greenium for bond b issued by country c at time t. The term $\mathcal{G}^{F}_{ct}$ is an issuer-wide fundamental commitment component: the value created when a green covenant changes the sovereign's continuation state. The term Λ_{bct} is a bond-specific residual wedge. The term $\mathcal{P}_{bct}$ captures pass-through or dilution of the fundamental benefit to all pari-passu claims, which reduces the relative green-conventional yield difference. The term $\mathcal{R}_{bct}$ captures measurement error from the empirical counterfactual. The structural model is about $\mathcal{G}^{F}_{ct}$, not the unconditional mean of $\mathcal{G}^{obs}_{bct}$.

This distinction matters because the sovereign greenium literature finds small and heterogeneous pricing effects. Sovereign greeniums are often much smaller than the variation in ordinary sovereign spreads and can be sensitive to the counterfactual curve or matched bond used (Ando et al., 2024; Panizza et al., 2025; Grzegorczyk and Wolff, 2022; D'Amico et al., 2025). That evidence is hard to reconcile with a simple story in which investors always pay a large premium for the green label. It is easier to understand once the observed greenium is decomposed into an issuer-wide fundamental component and bond-level residual wedges. It also changes the empirical task. The paper should not only ask whether the labelled bond trades rich; it should ask whether green-bond programs are credible, whether proceeds are additional, whether conventional bonds and CDS respond, and whether climate-risk states are the states in which commitment is most valuable.

The mechanism is a commitment mechanism. Climate risk can reduce future fiscal capacity through output losses, reconstruction needs, and infrastructure damage (Mallucci, 2022; Kling et al., 2021; Klusak et al., 2023). Adaptation investment can reduce these losses, but a sovereign that has already borrowed does not internalize all the benefits of future adaptation. Some benefits accrue to existing creditors through lower default risk and higher debt values. A green covenant can change the government's future incentives by creating an obligation to allocate, report, and verify eligible investment. If the covenant is credible and spending is additional, it can improve repayment capacity and lower issuer-wide default risk. If the program merely relabels past spending, or if enforcement is absent, the fundamental commitment value is zero.

The mechanism can be interpreted as a time-inconsistency problem in a sovereign-debt economy. Before debt is priced, the government would like to commit to adaptation because lenders value the resulting improvement in repayment capacity. After debt is issued, the government re-optimizes and does not fully internalize the gain accruing to existing creditors through higher debt values. A constrained adaptation-commitment planner would choose adaptation internalizing this borrowing-term effect, while still facing the same debt, default, climate, and resource constraints. The theory uses this planner as a benchmark for the missing commitment term; the quantitative analysis then compares regular-only borrowing, state-contingent green access, mandatory green issuance, and no-enforcement labels.

This commitment friction is related to, but distinct from, the standard debt-dilution problem in long-term sovereign debt. Debt dilution concerns future borrowing that reduces the value of legacy claims. The green-bond friction concerns future spending composition: after funds are raised, the government may underprovide adaptation even when adaptation would have improved ex-ante borrowing terms. Both are time-inconsistency problems, but they operate on different policy margins.

The empirical evidence is consistent with this decomposition. In the daily FactSet sample, curve-residual greeniums are close to zero and statistically weak. Individual matched-bond estimates using the Europe and Americas panels show near-zero yield greeniums and only modest asset-swap and z-spread richness. Benchmark-tenor comparisons are even more sensitive, reflecting the special liquidity and curve role of benchmark bonds. Public event studies around frameworks, second-party opinions, allocation reports, and impact reports show no large immediate bond-level repricing. Issuer-level CDS responses are economically small and become weaker once daily changes and global bond-market volatility are controlled for. At the same time, official allocation reports reveal large heterogeneity in adaptation content and in the share of proceeds allocated to prior-year versus current or forward-looking spending. The facts point away from a large unconditional label-premium story and toward a credibility-and-additionality story.

The quantitative model evaluates that story. It builds on the sovereign default tradition of Eaton and Gersovitz (1981), Arellano (2008), Hatchondo and Martinez (2009), and Chatterjee and Eyigungor (2012). The sovereign issues long-term defaultable debt, faces income and climate-risk shocks, accumulates adaptation capital, and chooses whether positive net issuance is regular or green. Green issuance creates an obligation stock. There is one fundamental price schedule for pari-passu debt; the label affects prices only through the continuation state it implies. The calibration uses Brazil as the baseline economy because the project has country-specific income, disaster, allocation, sovereign-risk, and green-issuance data. In the Brazil benchmark, green access lowers mean spreads from 398 to 270 basis points, reduces annual default frequency from 2.87 to 2.48 percent, and raises initial-state welfare by 1.29 percent relative to regular-only borrowing. These gains do not imply that all green debt is desirable: mandatory green issuance is welfare reducing because it removes fiscal flexibility.

The paper makes four contributions. First, it provides a no-arbitrage-safe framework for sovereign green-bond pricing. In a pari-passu sovereign setting, the fundamental price of default-risk improvements cannot be assigned only to the labelled claim. Second, it reframes the empirics by separating bond-level residual greeniums, issuer-wide default-risk responses, and credibility/additionality measures. This reframing matters because yield, asset-swap, and z-spread greeniums do not measure the same object. Third, it develops a quantitative sovereign default model in which green bonds are market-based fiscal commitment devices rather than assets with an exogenous green price wedge. The model links the empirical decomposition to welfare, default risk, debt capacity, and adaptation. Fourth, it studies policy. Green bonds are not universal cheap debt. They can be valuable commitment options when the covenant is credible and spending is additional, but high-debt and climate-vulnerable sovereigns may need guarantees, grants, concessional finance, or debt relief for the commitment technology to work.

The rest of the paper proceeds as follows. Section 2 discusses the related literature. Section 3 describes the institutional setting and the decomposition. Section 4 develops the commitment mechanism. Section 5 presents the empirical evidence. Sections 6 and 7 develop and calibrate the quantitative model. Sections 8 and 9 report the quantitative and policy results. Section 10 concludes.

2 Related Literature

This paper connects four literatures that usually speak to one another only indirectly: sovereign green-bond pricing, sustainable finance and investor demand, climate risk in sovereign debt, and commitment in quantitative sovereign-default models. The connection matters because each literature by itself gives an incomplete interpretation of the sovereign greenium. Green-bond pricing papers measure a relative yield wedge. Climate-sovereign-risk papers show that climate exposure can affect fiscal capacity and credit risk. Sovereign-default models explain how default risk and borrowing terms are jointly determined. Commitment models explain why governments may value constraints on future policy. The contribution here is to put those pieces together in a setting where the green label is attached to ordinary pari-passu sovereign debt.

The first strand estimates greeniums. In corporate and municipal markets, Zerbib (2019), Baker et al. (2018), Hachenberg and Schiereck (2018), Bachelet et al. (2019), Fatica et al. (2021), Tang and Zhang (2020), and Flammer (2021) show that labelled green bonds can trade differently from conventional bonds, but the estimates depend on issuer type, certification, investor clientele, and liquidity. Kapraun et al. (2021) emphasize credibility and external verification as determinants of the green-bond premium. Structural green-bond pricing models such as Agliardi and Agliardi (2019) and Agliardi and Agliardi (2021) formalize how climate-related risk and green claims can affect bond values, mainly in corporate settings. The sovereign case is different because the cash flow is usually not a project claim. The bond is backed by the sovereign's general repayment capacity, so a green covenant that changes default risk should affect the issuer's whole debt stock.

A newer literature studies sovereign greeniums directly. Ando et al. (2024) estimate sovereign greeniums using matched conventional bonds and highlight the German twin-bond design as a useful benchmark. Grzegorczyk and Wolff (2022) study European sovereign green bonds and document positive but heterogeneous greeniums. D'Amico et al. (2025) exploit Germany's twin-bond structure to separate a benchmark greenium from other spread components. Panizza et al. (2025) assemble a broad sovereign and quasi-sovereign sample and find that the greenium is statistically detectable but economically small, with important heterogeneity between advanced economies and emerging markets. Doronzo et al. (2021) discuss sovereign green bonds from the issuer perspective, while Cheng et al. (2024) show that sovereign green-bond debuts can foster domestic sustainable debt markets by improving issuance, liquidity, and verification standards. The empirical results are closest to this literature, but the interpretation is different. Rather than treating the measured sovereign greenium as the object to be explained, the paper interprets it as a residual after issuer-wide commitment value has passed through to other claims.

The second strand studies ESG preferences and sustainable-asset pricing. In models with tastes or mandates, investors may accept lower expected returns for assets with desirable environmental characteristics (Fama and French, 2007; Pástor et al., 2021). Investor surveys and climate-finance reviews document that climate risk and sustainability mandates are increasingly important for institutional investors (Krueger et al., 2020; Hong et al., 2020; Giglio et al., 2021). This preference channel is relevant for sovereign green bonds, and the decomposition in this paper allows for it through the bond-specific residual wedge. The point is not that investor demand is absent. It is that investor demand is not the only possible mechanism. A green covenant may also affect fundamentals if it changes future public spending, adaptation, or fiscal risk. The empirical distinction is subtle: a pure label-demand channel should be concentrated in the labelled security, while a fundamental default-risk channel should partly pass through to conventional bonds and CDS.

The third strand studies climate risk and sovereign credit. Cevik and Jalles (2020), Kling et al. (2021), and Beirne et al. (2021) provide empirical evidence that climate vulnerability and resilience are related to sovereign borrowing costs. Klusak et al. (2023) show that rising temperatures can affect sovereign credit ratings, while Zenios (2022), Boitan and Marchewka-Bartkowiak (2022), and Cappiello et al. (2025) study broader links between climate exposure, ratings, and sovereign debt risk. Mallucci (2022) is especially close to the quantitative side of this paper: he embeds natural-disaster risk in a sovereign-default model and shows that disaster risk can reduce market access and welfare, while disaster clauses can improve borrowing terms. This paper builds on the same logic that climate risk can be priced through sovereign default risk, but studies a different instrument. Green bonds do not automatically insure disaster states. They can matter if use-of-proceeds obligations change adaptation or transition investment enough to affect repayment capacity.

The fourth strand is the quantitative sovereign-debt literature. The model builds on the canonical incomplete-markets sovereign-default framework of Eaton and Gersovitz (1981), Aguiar and Gopinath (2006), and Arellano (2008), and on long-duration debt models such as Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012). Long-term debt is important because the value of future policy changes is capitalized into existing claims. This connects the paper to the debt-dilution literature. Hatchondo et al. (2016) show that future borrowing decisions can dilute outstanding debt and raise default risk; Arellano and Ramanarayanan (2012) and Aguiar et al. (2019) study how maturity interacts with lack of commitment. The green-bond friction is analogous but operates on the spending side. After debt is issued, a sovereign may underprovide adaptation relative to the commitment allocation because part of the benefit accrues to creditors through lower default risk and higher debt values.

The commitment interpretation also relates to fiscal rules and constrained discretion. Halac and Yared (2014) and Halac and Yared (2018) study optimal fiscal rules under persistent shocks, while Hatchondo et al. (2022) show how debt limits can reduce sovereign default premia. Green bonds are not fiscal rules in the usual sense: they do not cap aggregate debt. They constrain the use, reporting, and verification of a subset of financing. This makes them a partial and imperfect commitment technology. They can move the decentralized allocation toward a constrained commitment benchmark when the obligation is credible and spending is additional, but they can also reduce welfare when the obligation is too rigid. This is why the model distinguishes access to green borrowing from mandatory green issuance.

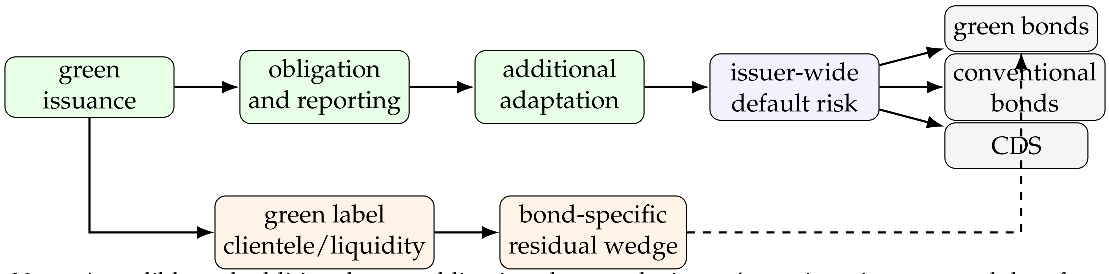
The paper's novelty is therefore not the claim that green bonds trade rich, nor the claim that climate risk matters for sovereign spreads. The contribution is to show that, for pari-passu sovereign debt, the observed bond-level greenium is a weak sufficient statistic for the value of the green covenant. A small measured greenium can be consistent with a meaningful issuer-wide commitment effect, with no fundamental effect, or with a mixture of fundamentals and residual label-demand wedges. Distinguishing these cases requires theory, bond-level pricing, issuer-risk evidence, public-program additionality measures, and a quantitative model of default and commitment.

3 Institutional Background and Decomposition

Sovereign green bonds combine ordinary sovereign debt with public expenditure commitments. A typical program contains a green-bond framework, an eligible-expenditure list, a second-party opinion or external review, allocation reports, and impact reports. These documents define what the issuer promises to finance and how it will report compliance. They do not usually create a senior claim on green assets or a separate default event for the green bond. The bond remains a claim on the sovereign.

This institutional structure is central. In a corporate project-finance setting, a green security might have claim-specific cash flows or collateral. In the sovereign setting, the bond's legal payoff is usually tied to the same issuer and default event as conventional debt. A green covenant can still matter, but its fundamental default-risk effect operates through the sovereign's future policy and repayment state. Once that state changes, the effect should be shared by all pari-passu claims with exposure to the same default event.

Figure 1: A sovereign greenium decomposes into issuer-wide commitment value and bond-specific residual wedges.



Notes: A credible and additional green obligation changes the issuer's continuation state and therefore default risk for all pari-passu claims. Separately, the green label can create a bond-specific residual wedge through clientele, liquidity, scarcity, or index effects.

3.1 Three Pricing Objects

The paper distinguishes three objects.

The first is the issuer-wide commitment component, $\mathcal{G}^F_{ct}$. It is the change in the sovereign's fundamental default-risk pricing caused by a credible green covenant that changes future policy. This is an issuer-level object. It can affect new green bonds, outstanding conventional bonds, benchmark bonds, and CDS spreads.

The second is the bond-specific residual wedge, Λ_{bct}. It captures forces that can make a green bond trade differently from a conventional bond even after issuer-wide risk is held fixed: label demand, ESG mandates, scarcity, liquidity, repo specialness, index inclusion, tax treatment, clientele effects, coupon differences, benchmark status, or idiosyncratic issue characteristics. This object is closest to what matched-bond and curve-residual greenium regressions estimate.

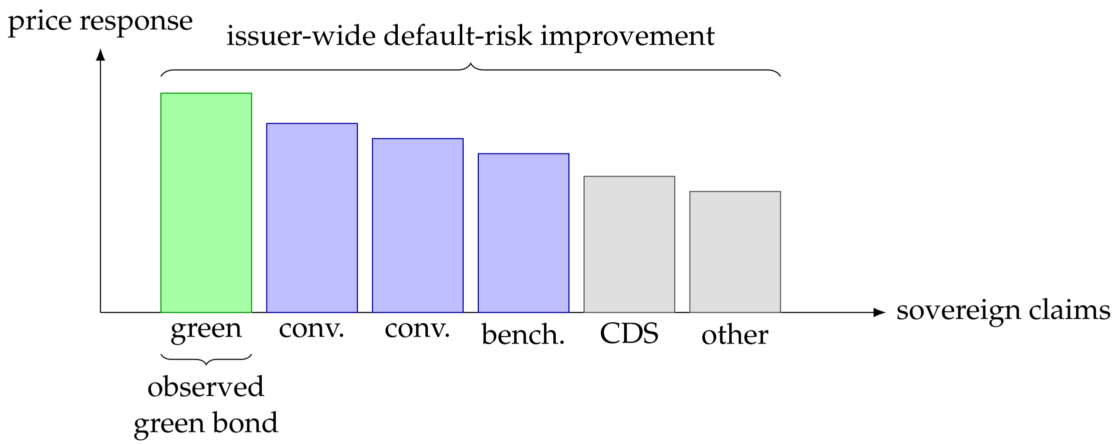
The third is the empirical measurement wedge, $\mathcal{R}_{bct}$. Sovereign green bonds rarely have perfect conventional twins. Yield, asset-swap spread, and z-spread comparisons differ because they net out different interest-rate, credit, liquidity, and optionality components. Benchmark bonds are special instruments. Matched conventional bonds can differ in coupon, age, amount outstanding, liquidity, or residual maturity. These differences create measurement error in the observed greenium.

Putting the pieces together,

$$\mathcal{G}_{bct}^{obs} = \mathcal{G}_{ct}^{F} + \Lambda_{bct} - \mathcal{P}_{bct} + \mathcal{R}_{bct},$$

where $\mathcal{P}_{bct}$ denotes pass-through of the issuer-wide fundamental value to other claims. The sign convention emphasizes that pass-through can make the relative green-conventional yield differential smaller: if both green and conventional bonds tighten after a credible green covenant, the green bond's own residual greenium need not be large.

Figure 2: Pass-through shares issuer-wide commitment value across pari-passu claims.



Notes: If a green covenant lowers issuer default risk, the fundamental benefit is shared across claims exposed to the same default event. The green bond's relative yield need not equal the issuer-wide commitment value.

3.2 Why Additionality Matters

The covenant has fundamental value only if it changes real or expected policy. A framework can be credible in the sense of being transparent and externally reviewed, but still have limited additionality if proceeds refinance past expenditures or relabel spending that would have happened anyway. Conversely, a current or forward-looking allocation to adaptation, resilience, transition investment, or fiscal-risk reduction is more relevant to the model because it can change future repayment capacity.

The empirical section therefore treats credibility and additionality as different mechanisms. Credibility concerns whether investors can observe, verify, and discipline the promise. Additionality concerns whether the promise changes real expenditure relative to the counterfactual. The model needs both: without credibility, obligations do not bind; without additionality, obligations do not change adaptation or fiscal capacity.

3.3 Implications for Empirical Design

The decomposition changes what should be measured. Bond-level greenium estimates are useful, but they estimate Λ_{bct} plus imperfect traces of issuer-wide value, not $\mathcal{G}^F_{ct}$ directly. Issuer-wide measures such as CDS spreads and conventional-bond spreads are closer to the fundamental default-risk component. Allocation reports, framework features, and additionality proxies identify whether the covenant has the economic content needed for $\mathcal{G}^F_{ct}$ to be nonzero.

The empirical design therefore asks three questions. First, how large are bond-level residual greeniums across yield, asset-swap, and z-spread measures? Second, do public green-bond events move issuer-wide default-risk measures? Third, do public documents reveal enough credibility and additionality for a commitment mechanism to be plausible?

4 Theory: Commitment, Pass-Through, and Additionality

This section develops the economic logic behind the empirical design and the quantitative model. The point of the simple model is not to reproduce the full recursive environment. It is to isolate three forces that any serious interpretation of sovereign greeniums must confront. First, pari-passu sovereign claims cannot have different fundamental default-risk prices merely because one bond is labelled green. Second, green commitments can matter if they change future policy in a way that lowers default risk. Third, the value created by such a commitment is issuer-wide, so a bond-level greenium can be small even when the commitment has real value.

4.1 Payoff Equivalence

Lemma 4.1 (Pari-passu payoff equivalence)**.** *Fix a state s. Let bonds a and b have the same issuer, currency, maturity, coupon structure, seniority, tax treatment, recovery rule, and default event. Suppose lenders price fundamental cash flows competitively under a common stochastic discount factor and there are no bond-specific label preferences, liquidity premia, clientele effects, index effects, tax differences, or recovery differences. Then*

$$q_a(s) = q_b(s).$$

The lemma rules out a model in which two otherwise identical pari-passu claims have different fundamental default-risk prices in the same state. A green bond can have fundamental value only if the covenant changes the continuation state, the cash-flow contract, taxes, recovery, liquidity, or investor constraints. The proof is immediate from state-contingent pricing and is given in Appendix A.

4.2 Adaptation and Default Risk

The next step is to show what it means for the covenant to change the continuation state. The simplest case is adaptation. Climate adaptation raises future repayment resources by protecting output, reducing reconstruction needs, or preserving tax capacity after a disaster. This does not mean that all adaptation is priced by bond markets. It is priced only to the extent that it changes the default boundary.

There are three dates. At date 0, the sovereign issues face value B. At date 1, it chooses adaptation investment i. At date 2, repayment resources are

$$Y_2 = y + A(i) + \varepsilon,$$

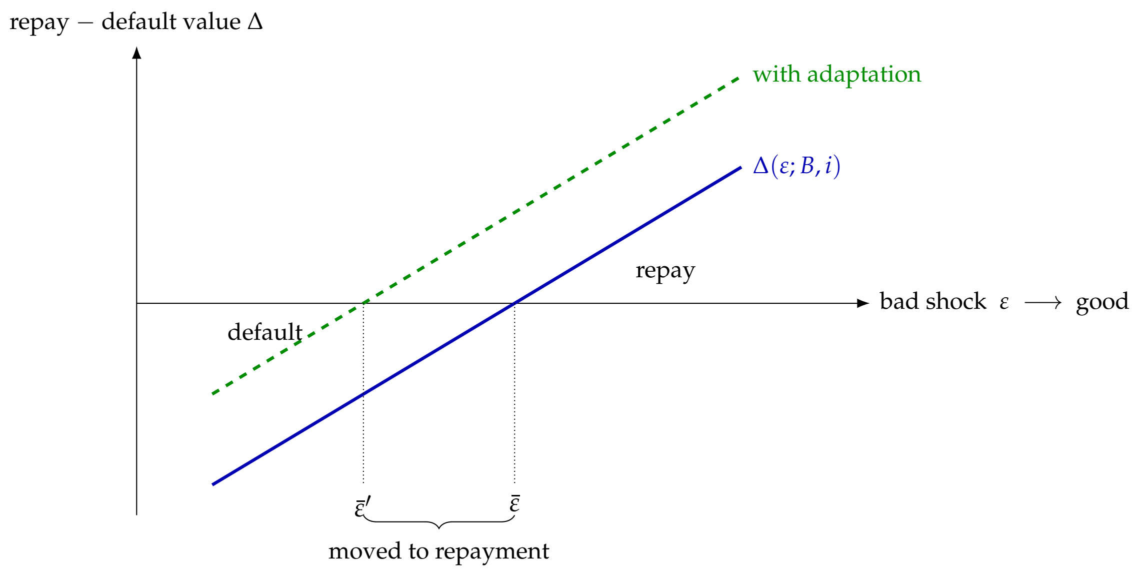
where $A'(i) > 0$, $A''(i) \leq 0$, and ε has cdf F and density f. Default is endogenous. Let the repayment and default payoffs be $U^R(\varepsilon; B, i)$ and $U^D(\varepsilon; i)$, and define

$$\Delta(\varepsilon; B, i) = U^R(\varepsilon; B, i) - U^D(\varepsilon; i).$$

Assumption 1 (Single crossing and adaptation relevance)**.** *For each (B, i), Δ is continuous and strictly increasing in ε. There is a unique threshold $\bar{\varepsilon}(B, i)$ satisfying $\Delta(\bar{\varepsilon}; B, i) = 0$. At the threshold, $\Delta_B < 0$ and $\Delta_i > 0$.*

The condition $\Delta_i > 0$ is the primitive economic restriction that adaptation improves repayment capacity relative to default value. This can hold because adaptation protects taxable output, reduces reconstruction needs in repayment, preserves market access, or raises creditor-relevant fiscal capacity more than autarky value.

Figure 3: Default is a single-crossing decision; adaptation lowers the threshold.



Notes: The sovereign repays when the value of repaying exceeds the value of defaulting, i.e. when the difference $\Delta(\varepsilon; B, i)$ is positive. Under Assumption 1, Δ is increasing in the shock ε and crosses zero once, at the default threshold $\bar{\varepsilon}(B, i)$: the sovereign defaults for $\varepsilon < \bar{\varepsilon}$ and repays for $\varepsilon \geq \bar{\varepsilon}$. Adaptation raises repayment resources, shifting Δ up and lowering the threshold to $\bar{\varepsilon}'$, so states in $[\bar{\varepsilon}', \bar{\varepsilon})$ move from default to repayment.

Under Assumption 1, the sovereign repays if $\varepsilon \geq \bar{\varepsilon}(B, i)$, with

$$\bar{\varepsilon}_B(B, i) > 0, \qquad \bar{\varepsilon}_i(B, i) < 0.$$

With zero recovery and risk-neutral lenders, the fundamental price is

$$q(B, i) = \frac{1}{1+r}\left[1 - F(\bar{\varepsilon}(B, i))\right].$$

Hence

$$q_i(B, i) = -\frac{1}{1+r} f(\bar{\varepsilon}(B, i))\bar{\varepsilon}_i(B, i) > 0.$$

Adaptation affects prices only through probability mass near the repayment boundary. This is the default-risk elasticity channel. It explains why the same adaptation project can have different pricing effects across sovereign states: the price response is large when adaptation moves states across the repayment/default margin and small when the issuer is either very safe or so distressed that adaptation does not restore repayment.

4.3 Underinvestment, Time Inconsistency, and Green Covenants

The pricing channel creates a time-inconsistency problem. Before debt is sold, the sovereign would like to promise future adaptation if that promise lowers borrowing costs. After debt is outstanding, part of the gain from adaptation accrues to legacy creditors through higher bond values. A discretionary government therefore may choose too little adaptation relative to the commitment allocation, even if it values climate benefits directly.

After debt is sold, a discretionary government chooses adaptation by maximizing

$$G(i; B) = v(i; B) - C(i),$$

where $C_i > 0$ and $C_{ii} > 0$. A date-0 planner who can commit future adaptation internalizes the effect of adaptation on borrowing proceeds:

$$P(i; B) = v(i; B) - C(i) + Bq(B, i).$$

If both objectives are concave and interior, the discretionary and commitment choices satisfy

$$v_i(i^R; B) - C_i(i^R) = 0,$$

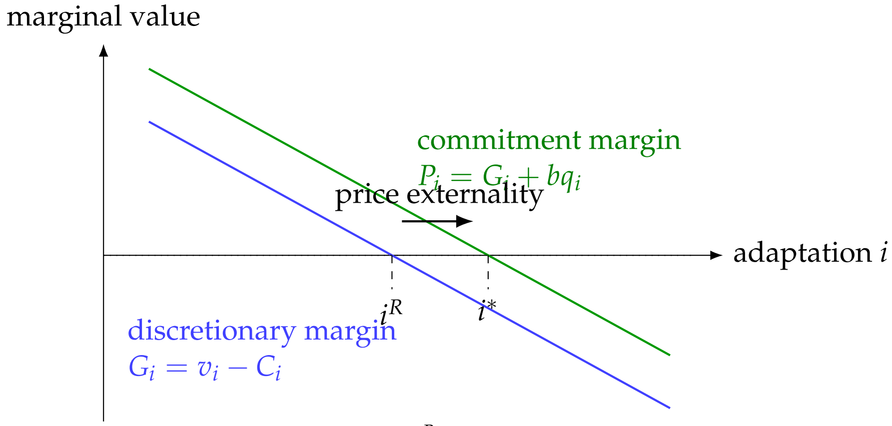
and

$$v_i(i^P; B) - C_i(i^P) + Bq_i(B, i^P) = 0.$$

The term $Bq_i(B, i)$ is the missing commitment term. This is not a claim that governments ignore climate benefits: those are in v_i. The wedge is ex ante versus ex post. Before debt is priced, the government would like to commit to high adaptation because lenders pay more for safer debt. After debt is outstanding, part of the gain from adaptation accrues to creditors through lower default risk and higher debt values. The discretionary choice $i^R(B)$ is therefore below the constrained commitment-planner choice $i^P(B)$ whenever $Bq_i(B, i^R) > 0$.

A green covenant can partially implement the missing commitment term. It creates an obligation m and a compliance technology. Let $a \in [0, 1]$ denote additionality: the fraction of the promised

Figure 4: Discretionary underinvestment in adaptation.



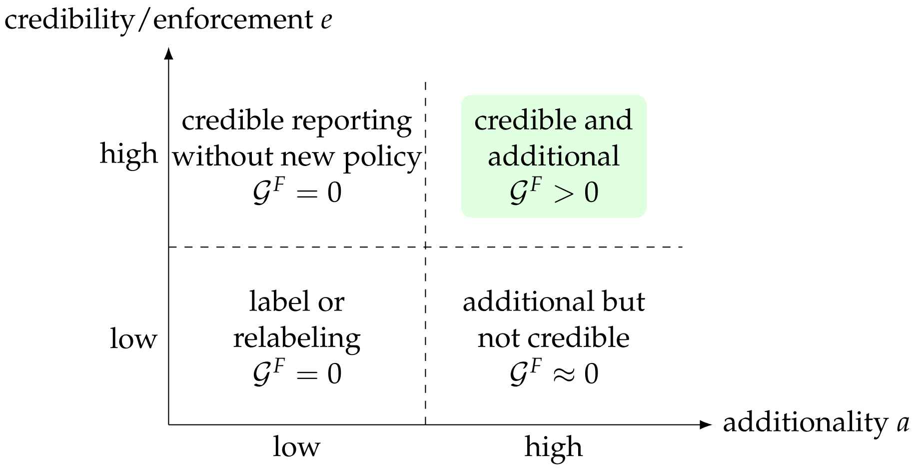
Notes: The discretionary government chooses i^R, where the direct adaptation margin is zero. The commitment planner internalizes bq_i, the effect of adaptation on borrowing proceeds, and therefore chooses $i^* > i^R$.

green obligation that changes real adaptation relative to the regular-debt counterfactual. Let $e \geq 0$ denote enforcement or credibility. A reduced-form green adaptation response is

$$i^G(B, m, a, e) \geq i^R(B),$$

with strict inequality only if both the covenant is credible and the expenditure is additional. A transparent but purely retrospective allocation has $a = 0$. A label with no enforcement has $e = 0$.

Figure 5: Commitment value requires both credibility and additionality.



Notes: A green label with no enforcement or no marginal spending response has no fundamental default-risk value. The commitment channel operates when the program both binds and changes real adaptation or fiscal capacity.

Proposition 4.2 (Commitment requires credible additionality)**.** *If the green label does not change real policy, enforcement, resources, recovery, liquidity, taxes, investor constraints, or the feasible set, then* $i^G(B, m, a, e) = i^R(B)$ *and the issuer-wide fundamental commitment value is zero. If* $i^G(B, m, a, e) > i^R(B)$*, then the covenant raises the issuer-level fundamental price:*

$$q(B, i^G) > q(B, i^R).$$

The proof, in Appendix A, uses the price derivative above. The proposition is the formal reason for coding public allocation reports, adaptation shares, current versus retrospective spending, assurance, and reporting quality in the empirical section. These variables are not ancillary controls around the greenium regression. They are observable proxies for whether a and e are likely to be large enough for the commitment channel to operate.

4.4 Markov Equilibria and the Commitment-Planner Benchmark

The simple model is a static representation of the recursive objects used later. Let $i^R(B)$ denote adaptation in the regular-only discretionary economy, $i^G(B, m, a, e)$ denote adaptation induced by a green covenant with obligation m, additionality a, and enforcement e, and $i^P(B)$ denote adaptation under a constrained commitment planner. The natural partial-implementation relationship is

$$i^R(B) \leq i^G(B, m, a, e) \leq i^P(B).$$

The first inequality is strict only if the covenant is credible and additional. The second inequality can fail if the green obligation is too rigid, poorly timed, or forces investment beyond the constrained planner's preferred allocation. The planner benchmark is constrained: it faces the same resource, debt, default, climate, and pricing constraints as the sovereign economy, but it can commit the future adaptation choice directly.

Definition 4.3 (Equilibrium and planner objects)**.** *The regular-only Markov equilibrium is the no-green, no-commitment equilibrium in which the sovereign chooses default, debt, and adaptation state by state and lenders price debt competitively. The green-access Markov equilibrium is the same equilibrium concept with an added green issuance option and inherited obligation state. The constrained adaptation-commitment planner is a benchmark in which future adaptation can be committed directly, while the same default, debt, climate, and resource constraints remain.*

This language clarifies the role of green bonds. They are not assumed to be cheaper debt. They are an imperfect implementation technology for moving a Markov-discretionary sovereign allocation toward a constrained commitment allocation. The quantitative model below is exactly this decentralized object: lenders price defaultable long-term debt competitively, while the sovereign re-optimizes over default, debt, adaptation, and green obligations state by state.

4.5 Pass-Through to Pari-Passu Claims

Let K index the sovereign's outstanding pari-passu claims. Claim k has cash flow $X_k(h)$ in history h, all exposed to the same default event. A green covenant changes the continuation state from s^R to s^G. The fundamental valuation change for claim k is

$$\Delta q_k = \mathbb{E}\left[M(h)\left(X_k(h; s^G) - X_k(h; s^R)\right)\right].$$

If the only change is a reduction in default probability, all claims exposed to that default event receive a fundamental benefit, scaled by duration, coupon, recovery, and cash-flow timing.

Proposition 4.4 (Issuer-wide pass-through)**.** *Suppose a green covenant changes the issuer contin-*

uation state and lowers default risk. Then every pari-passu claim exposed to the same default event experiences a fundamental price effect. The bond-level greenium between a green bond and a conventional bond identifies the full issuer-wide commitment value only under the knife-edge condition that no fundamental value passes through to the conventional comparison bond.

The proof is given in Appendix A. This is the key asset-pricing implication. A small green-conventional yield differential does not imply a small issuer-wide commitment value. It may indicate that the value is shared across the debt stock. This is why the empirical section studies matched greeniums, conventional-bond responses, and CDS responses rather than treating the green bond's relative yield as the only outcome.

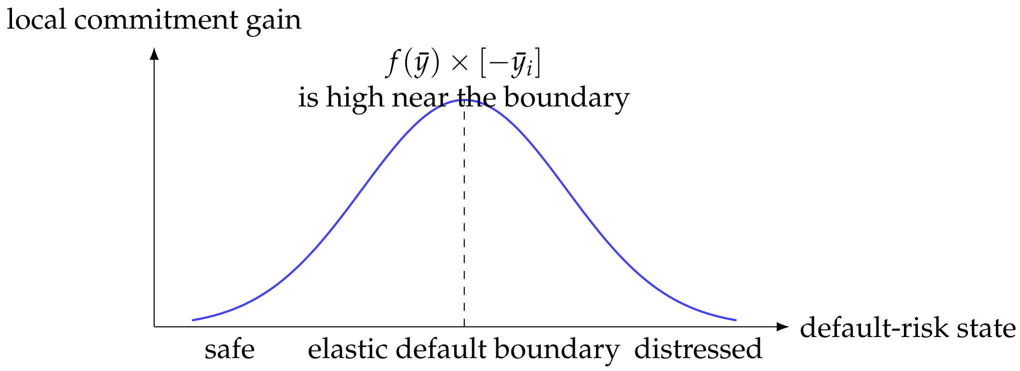
4.6 State-Contingent Value and Welfare

The local commitment gain from an additional unit of adaptation is proportional to

$$f(\bar{\varepsilon}(B,i))[-\bar{\varepsilon}_i(B,i)].$$

It is large when default risk is material and elastic to adaptation. It can be small for safe issuers because little probability mass is near default, and small for deeply distressed issuers if adaptation does not move the boundary enough or if future obligations are not credible.

Figure 6: Commitment value is state dependent, not mechanically monotone in spreads.



Notes: A green commitment has little default-risk value for very safe issuers because little mass is near default. It can also have limited value for deeply distressed issuers if adaptation cannot move the repayment boundary or if the covenant is not credible.

Prediction 4.5 (State-contingent commitment)**.** *Issuer-wide commitment value is highest when default risk is material, adaptation materially improves repayment capacity, the covenant is credible, and spending is additional. The relationship between observed bond-level greeniums and sovereign spreads need not be monotone.*

Finally, price gains and welfare gains are different. A covenant can lower default risk but impose costly spending obligations in states where fiscal flexibility is valuable. The welfare question therefore cannot be answered from greeniums alone; it requires a model of fiscal choices, default risk, and the cost of restricting future policy.

Prediction 4.6 (Option value of green access)**.** *Green access can dominate regular-only borrowing by giving the sovereign a state-contingent commitment option. Mandatory green issuance can be worse than* *green access because it removes fiscal flexibility.*

5 Empirical Evidence

The empirical section is organized around the decomposition in Section 4. A small observed greenium is not, by itself, evidence that green bonds are irrelevant. It can mean that the covenant has little value, but it can also mean that the value is issuer-wide and therefore passes through to conventional bonds and CDS. Conversely, a positive bond-level greenium need not be fundamental; it may reflect liquidity, scarcity, clientele demand, or benchmark status. The empirical strategy therefore separates five objects: the relative price of the labelled bond, the response of issuer-wide risk, the credibility of the commitment, the additionality of spending, and the state dependence created by climate risk.

5.1 Data and Empirical Objects

The analysis combines four types of data. First, daily FactSet ISIN data provide yields, prices, and duration for sovereign green bonds and nearby conventional bonds. Second, monthly Europe and Americas sovereign-bond panels provide yields, asset-swap spreads, z-spreads,[1] bid and ask quotes, maturity, ratings, and outlooks. Third, public debt-management-office documents provide framework dates, second-party opinions, allocation reports, impact reports, use-of-proceeds categories, adaptation shares, and accounting timing. Fourth, issuer-risk and climate files provide 5-year CDS spreads, the MOVE index, FactSet/LSEG country climate-risk variables, ND-GAIN vulnerability and readiness, and EM-DAT realized climate-disaster exposure.

Table 1 reports the working sample. The daily ISIN panel contains 105 conventional ISINs and 196 green ISINs with usable yield observations. The curve-residual design uses 30,204 green-bond-date observations. The matched-pair design contains 117 green-conventional pairs. The public-source event corpus contains 29 dated events, and 10 countries have usable 5-year CDS coverage. The matched monthly panel is the main pricing object because it lets us compare yields, ASW, and z-spreads inside the same green-conventional pair. That comparison is essential: if the sign of the greenium changes across measures, the object is not a clean sufficient statistic for structural commitment value.

[1] Three relative-pricing measures recur throughout. The *yield spread* is the difference in yield to maturity between the green bond and its conventional comparator; it is the simplest measure but embeds coupon, duration, and rate-curve effects. The *asset-swap (ASW) spread* is the spread over the floating swap benchmark earned by holding the bond and swapping its fixed coupons to floating; it is the market-standard relative-value measure, computed under a par approximation. The *z-spread* (zero-volatility spread) is the constant spread added to every point of the swap zero-coupon curve that equates the discounted present value of the bond's cash flows to its market price; by discounting each cash flow at the matching spot rate it accounts for the term-structure shape and the bond's actual price, and is more precise than the ASW. ASW and z-spread coincide for a par bond on a flat curve and diverge otherwise; both strip out more rate and coupon variation than the raw yield spread, but both still embed liquidity, benchmark status, and modeling choices. We report all three because no single measure is a clean credit spread and, as the results below show, the sign and magnitude of the greenium can change across them—itself evidence that the bond-level greenium is not a clean structural object.

Table 1: Empirical coverage: pricing, issuer-risk, and program-design evidence

Object	Count	Unit
Regular bonds with daily YTM	105	ISINs
Green bonds with daily YTM	196	ISINs
Matched green-regular pairs	117	pairs
Curve-residual observations	30,204	ISIN-days
Curve-residual green bonds	55	ISINs
Public green-bond events	29	events
Audited public-source issuers	18	issuers
Countries with usable 5Y CDS	10	countries

Notes: The table summarizes the empirical objects used to separate bond-level greeniums, issuer-wide risk responses, and credibility/additionality mechanisms. Raw files remain in their original directories; all counts are generated from scripts in `empirics_jie/src`.

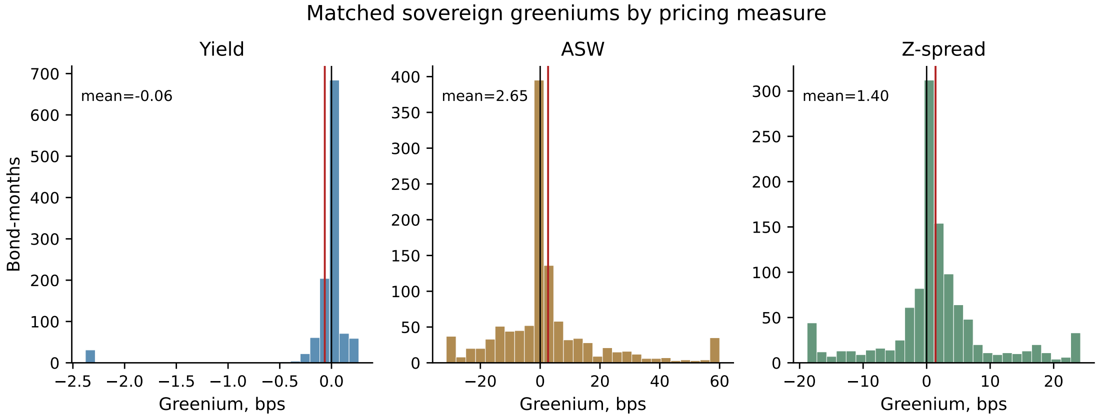
5.2 How Large Is the Bond-Level Greenium?

The first empirical question is deliberately narrow: how much richer is the labelled bond than a comparable conventional bond from the same issuer? For a green bond b, define the bond-level residual greenium as

$$\widehat{\mathcal{G}^{obs}}_{bct} = \widehat{y}^{R}_{bct} - y^{G}_{bct},$$

where $\widehat{y}^{R}_{bct}$ is the predicted conventional yield from a same-country conventional curve or matched conventional bond. Positive values mean that the green bond trades at a lower yield than the conventional counterfactual. Analogous residuals are computed for ASW and z-spread measures.

Figure 7: Observed greeniums are centered near zero and depend on the pricing measure



Notes: The figure reports winsorized histograms for matched bond-month greeniums using the two-year maturity-caliper sample. Positive values mean that the green bond is richer than the matched conventional bond. The vertical red line is the sample mean. The dispersion and measure dependence reinforce that the bond-level greenium is a residual pricing object, not a direct estimate of issuer-wide commitment value.

Figure 7 shows the first fact: the distribution of matched greeniums is centered close to zero, and the center differs across pricing measures. This is not only a statistical issue. Yield residuals mix rate exposure, duration, coupon, and curve effects. ASW and z-spread residuals are closer to spread pricing, but they still embed liquidity, benchmark status, and modeling assumptions. The disagreement across measures is therefore evidence in itself: the observed bond-level greenium is a noisy residual wedge, not a direct measure of the value created by a green covenant.

Table 2: Daily greenium estimates are small and statistically weak

Estimator	Obs.	Clusters	Mean	S.E.	p-value
Curve residual, daily	30,204	55	0.59	1.10	0.592
Curve residual, equal bond	55	55	-3.50	2.55	0.171
Matched pair, daily	42,367	76	-3.17**	1.47	0.031
Matched pair, equal pair	76	76	-3.75**	1.51	0.013

Notes: Greenium is measured in basis points as predicted conventional yield minus observed green yield. Positive values mean the green bond trades at a lower yield. Daily specifications cluster by green bond or matched pair; equal-weight specifications first average within bond or pair. The preferred curve-residual estimate is close to zero, which motivates looking beyond the labelled bond's yield residual.

Table 2 shows the same fact in the daily curve-residual design. The preferred curve-residual daily estimate is 0.59 basis points with a standard error of 1.10. Equal-weighting green bonds gives a negative mean, and matched-pair daily estimates are also negative on average. These magnitudes are small relative to ordinary sovereign-spread movements.

Table 3: Matched-bond greeniums differ across yields, ASW, and z-spreads

Measure	Obs.	Green ISINs	Countries	Mean bps	Median bps
Yield	1140	29	12	-0.06	0.01
ASW	1134	28	12	2.23	0.54
z-spread	1134	28	12	1.10	0.87

Notes: Monthly green bonds are matched to same-country, same-currency conventional sovereign bonds in the Europe and America individual-bond panels by closest remaining maturity, requiring a maturity gap no larger than two years. Positive values mean the green bond trades tighter than the matched conventional bond. The yield greenium is essentially zero, while ASW and z-spread greeniums are positive but modest, consistent with the paper's measurement decomposition.

Table 3 uses the monthly individual-bond panels. The matched-bond yield greenium is essentially zero; ASW and z-spread greeniums are positive but modest. This is the paper's baseline empirical fact. Sovereign green bonds do not display a large unconditional residual premium. The appendix shows that benchmark-bond residuals, maturity calipers, controls, and leave-one-country-out estimates do not overturn this conclusion. The rest of the section therefore asks whether the small residual premium is hiding issuer-wide pass-through or weak additionality.

5.3 Does Green News Pass Through to Issuer Risk?

The model's commitment component is issuer-wide. If a green-bond program credibly changes future adaptation or fiscal capacity, the response need not be concentrated in the green bond itself. It can pass through to conventional bonds and CDS because default risk is common across the sovereign's liabilities. This is why the event evidence is interpreted as a pass-through check rather than as a standard announcement test for a labelled-asset premium.

Table 4 reports event-study evidence from the expanded 5-year CDS panel. Framework and second-party-opinion events are followed by CDS declines of 0.64 basis points in 30-day win-

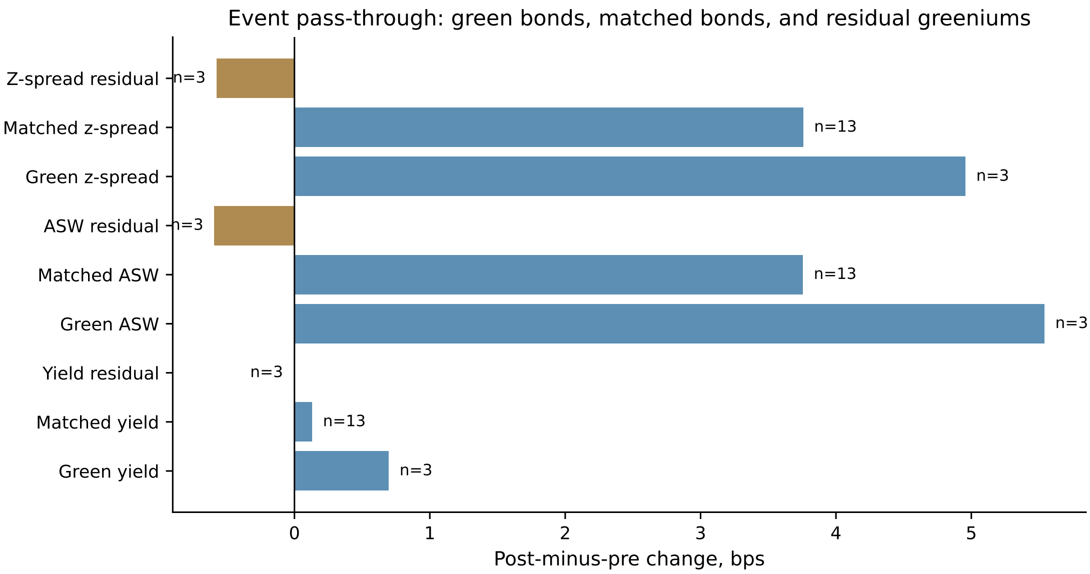
Table 4: Public green-bond events generate only modest CDS repricing

Event group	Window	Events	Post-pre bps	SE
Framework/SPO	30	9	-0.64	0.16
Framework/SPO	60	9	-1.34	0.48
Reporting	30	11	0.36	0.96
Reporting	60	11	-0.12	1.00
All public events	30	24	-0.12	0.45
All public events	60	24	-0.51	0.50

Notes: The table uses the cleaned daily 5-year CDS panel in `data/raw/CDS`. Each event contributes the difference between post-event and pre-event average CDS spreads within the window. The negative framework/SPO responses point toward issuer-risk pass-through, but the magnitudes are small and the comparison is descriptive because event timing may be anticipated and windows can overlap.

dows and 1.34 basis points in 60-day windows. These estimates point in the direction of modest issuer-risk repricing, but the magnitudes are economically small. Appendix B shows that the result is weaker when using daily CDS changes and MOVE controls, which argues against a large unanticipated announcement effect.

Figure 8: Public green-bond events move green and conventional bonds together



Notes: Bars are six-month post-minus-pre changes around audited public green-bond events, averaged at the event level. Residual rows are the corresponding green-minus-counterfactual greenium objects. Comparable movements in matched conventional bonds are consistent with issuer-wide pass-through and caution against reading event responses only through the labelled bond's residual spread.

Figure 8 compares public green-bond events across green bonds, matched conventional bonds, and the residual greenium. The residual rows are small relative to movements in green and matched conventional spreads. This matters for interpretation. If public green-bond news mostly moved the labelled bond through a special-clientele channel, the residual greenium would move strongly relative to the matched bond. Instead, pass-through to conventional bonds is often comparable. The event evidence is small-N, so it should not be read as a clean causal announcement design. Its contribution is more specific: it shows that green-bond news is not naturally summarized by the labelled bond's residual yield alone.

5.4 Credibility, Additionality, and Program Design

The theory requires more than a green label. A covenant creates fundamental value only if it is credible and changes real or expected fiscal choices. Public documents therefore enter the empirical analysis as mechanism variables. We code whether the issuer publishes a framework, second-party opinion, external assurance, allocation report, impact report, machine-readable allocation or impact data, taxonomy alignment, adaptation/resilience content, and quantitative impact metrics. We also parse official allocation files into mitigation, adaptation, nature, and accounting-timing categories when the source documents permit it. These variables are imperfect, but they are closer to the primitives in Proposition 4.2 than the bond yield residual alone.

Table 5: Official allocation reports reveal heterogeneous adaptation content and accounting timing

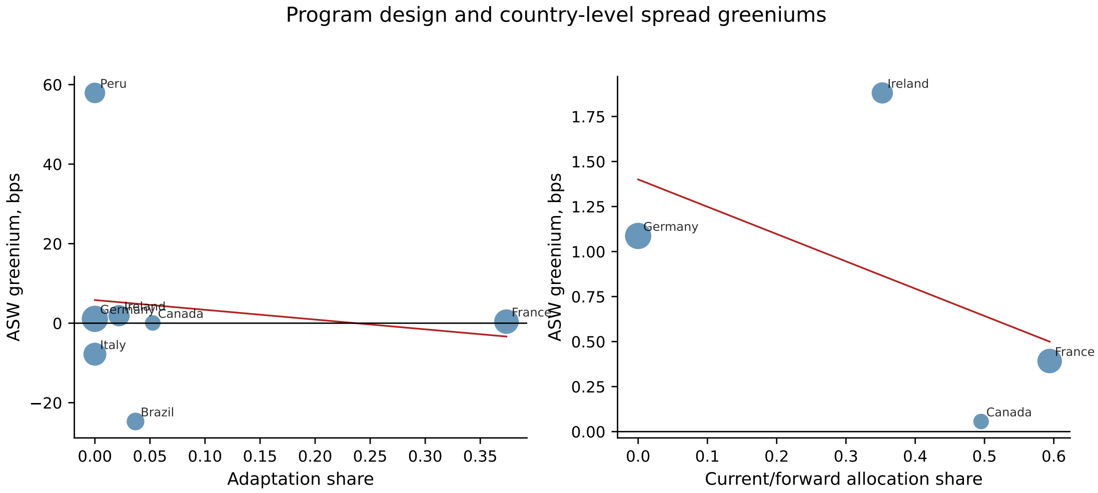
Issuer	Rows	Reported amount	Adapt.	Mitig.	Prior/refi	Current/fwd
Austria	8	6232.3 EUR mn	3.5%	87.8%	47.2%	52.8%
Belgium	329	19383.7 EUR mn	0.0%	98.4%	41.2%	58.8%
Brazil	72	5295.0 BRL mn	3.7%	3.8%	–%	–%
Canada	30	8725.7 CAD mn	5.3%	57.1%	50.5%	49.5%
Chile	4	2440.3 USD mn	0.0%	95.3%	44.8%	55.2%
Denmark	6	31985.2 DKK mn	0.0%	100.0%	92.4%	7.6%
France	65	8419.3 EUR mn	37.4%	89.7%	40.6%	59.4%
Germany	74	22449.0 EUR mn eligible	0.0%	75.8%	100.0%	0.0%
Ireland	81	5001534.3 EUR thousands	2.2%	66.2%	64.8%	35.2%
Italy	6	13360.8 EUR mn	0.0%	71.5%	–%	–%
Netherlands	4	4272.0 EUR mn	20.3%	100.0%	50.0%	50.0%
New Zealand	234	9807268242.3 NZD	7.0%	51.5%	34.4%	65.6%
Peru	3	3460.0 PEN mn	0.0%	0.0%	–%	–%
Spain	8	7121.0 EUR mn	10.2%	45.1%	80.7%	19.3%
UK	78	25164.7 GBP mn	14.2%	91.4%	8.4%	90.8%

Notes: Reported amounts use each issuer's own currency and source definition. Germany reports potential eligible expenditure rather than final allocation in the parsed file. Category shares are not mutually exclusive when a row spans multiple green categories. Prior/refi and current/fwd are accounting shares from official allocation reports where available; blanks mean the official source corpus does not yet provide a quantitative split. The variation in adaptation and timing is the empirical counterpart of the model's additionality margin.

Table 5 shows that sovereign green programs differ sharply in both composition and accounting timing. Some programs have substantial adaptation or resilience content; others are dominated by mitigation, transport, energy, or efficiency. Some allocate proceeds to prior-year or refinancing expenditure; others are more current or forward-looking. This distinction is central for the model. A retrospective allocation can be transparent and credible as accounting, but it is a weaker proxy for marginal adaptation investment than a current or forward-looking allocation.

Figure 9 links these program variables to country-level ASW greeniums. The relationship is not monotone. Countries with more adaptation content or more current/forward-looking allocation do not mechanically have larger bond-level spread greeniums. This is informative, not disappointing. It suggests that public-document quality and accounting labels are not sufficient statistics for market value. The market appears to price a mixture of credibility, marginal fiscal impact, liquidity, country risk, and investor demand. The appendix reports formal mechanism

Figure 9: Observable program design is not a sufficient statistic for greenium size



Notes: Each dot is an issuer-country mean. Dot size is proportional to the square root of matched bond-month observations. Program shares are parsed from official allocation and impact reports. The weak unconditional relationship is consistent with the paper's interpretation that credibility, additionality, liquidity, country risk, and investor demand are jointly reflected in observed greeniums.

regressions; they are imprecise and should be read as descriptive cross-country evidence rather than causal estimates because many program variables are issuer-level characteristics. The quantitative model takes this lesson seriously by targeting green issuance and adaptation intensity, while leaving the observed greenium as an untargeted validation object.

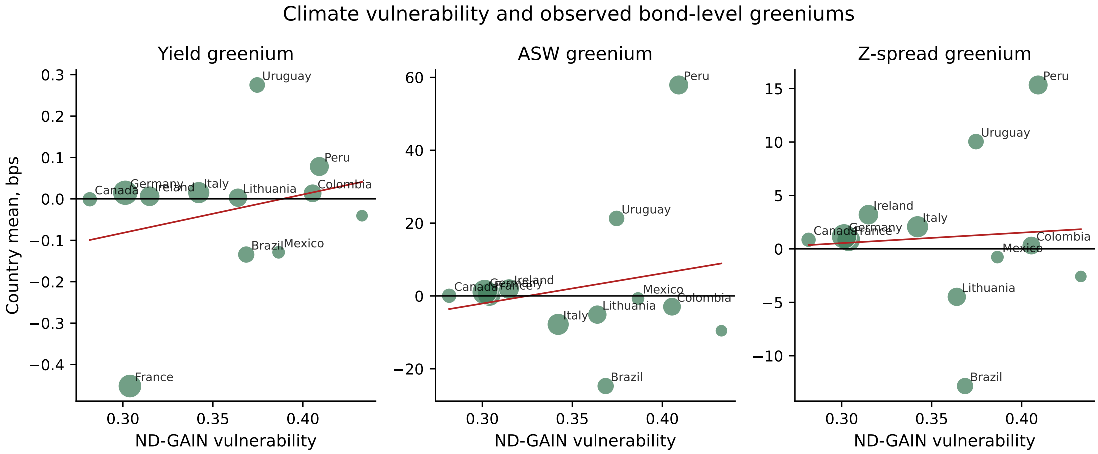
5.5 Climate Risk and Disaster Exposure

The final empirical block asks whether climate exposure strengthens the green-bond pricing effect. The model predicts state dependence, but not a simple monotone relationship between climate risk and observed greeniums. Adaptation is valuable when climate risk is high, but the same states can also raise borrowing needs and default risk. The observed residual greenium is therefore the net of a demand component, an issuer-risk component, and a supply/fiscal-stress component.

Figure 10 shows that ND-GAIN vulnerability does not map cleanly into larger observed greeniums. The slope and sign depend on whether the pricing object is a yield, ASW, or z-spread residual. This reinforces the paper's measurement point. Climate risk may raise the value of a credible adaptation commitment, but the bond-level residual can remain small if the commitment passes through to all bonds, if green issuance finances existing expenditure, or if disaster risk increases fiscal stress.

Appendix B reports two additional climate tests. First, forward-looking FactSet/LSEG climate-risk variables and lagged EM-DAT disasters do not overturn the near-zero average greenium once liquidity, ratings, and year effects are included. Second, contemporaneous EM-DAT disaster shocks interacted with pre-existing program design do not produce a robust increase in green-bond richness. Those results discipline the quantitative model: the calibration should not target a large unconditional greenium or a large disaster-induced residual premium. It should

Figure 10: Climate vulnerability does not translate mechanically into larger observed greeniums



Notes: Each dot is an issuer-country mean. ND-GAIN vulnerability is higher for more vulnerable countries. The fitted lines are descriptive cross-country slopes. The figure is consistent with the model's state-contingent prediction: climate need can raise the value of adaptation while also increasing fiscal stress and default risk.

instead match a small observed greenium together with modest issuer-wide pass-through and heterogeneous program credibility/additionality.

5.6 Empirical Takeaways

The empirical evidence delivers five facts for the quantitative model. First, the unconditional sovereign greenium is small and sensitive to the pricing measure. Second, ASW and z-spread residuals show somewhat more richness than raw yields, but the magnitude remains small relative to sovereign-risk variation. Third, public green-bond events do not generate large immediate issuer-wide repricing, and matched conventional bonds often move with green bonds. Fourth, credibility and additionality vary substantially across sovereign programs, and observable document quality alone does not generate a stable bond-level premium. Fifth, climate vulnerability and realized disasters matter only conditionally and with measure-dependent signs. These facts discipline the model in a particular way. The model should not manufacture a large unconditional bond-level greenium. It should instead quantify how much issuer-wide commitment value can coexist with small relative greeniums, modest pass-through, and heterogeneous additionality.

6 Quantitative Model

The quantitative model measures the issuer-wide commitment component that bond-level greeniums do not identify directly. It is a recursive sovereign default model with long-term debt, income risk, climate risk, adaptation capital, and green-obligation states. The structure deliberately separates the economics of the paper from an exogenous label-price wedge. Green debt can become valuable only because the obligation state changes future adaptation incentives, resources, and default risk. The central no-arbitrage restriction is built in: there is one fundamental price schedule for pari-passu sovereign debt.

6.1 State and Shocks

Time is discrete. The sovereign state is

$$s = (y, x, g, B, m),$$

where y is output, x is climate stress, g is adaptation capital, B is long-term defaultable debt, and m is the outstanding stock of green obligations. The first two states describe the shock environment. The last three describe the endogenous fiscal position inherited by the government. Output and climate stress follow finite Markov chains with transition matrices P_y and P_x.

6.2 Climate Damages and Adaptation

Effective repayment resources before debt service are

$$\tilde{y}(y, x, g) = y[1 - D(x, g)] - \Phi(x, g),$$

where $D_x > 0$, $D_g < 0$, $\Phi_x > 0$, and $\Phi_g < 0$. The first term captures output damages. The second captures fiscal losses such as reconstruction, emergency response, and infrastructure repair. Adaptation capital evolves according to

$$g' = (1 - \delta_G)g + i,$$

where $i \geq 0$ is adaptation investment. The baseline discretization uses this linear accumulation law and allows investment to be costly through the convex implementation cost in the resource constraint below.

6.3 Debt and Green Obligations

Debt is a long-term defaultable bond. Each unit of outstanding debt pays coupon κ and matures with probability δ_B. If the government repays, legacy debt service is $(\kappa + \delta_B)B$. The government chooses next-period debt B'. Net issuance is

$$\ell = B' - (1 - \delta_B)B, \qquad \ell^+ = \max\{\ell, 0\}.$$

The government also chooses an issuance label $J \in \{R, G\}$. If positive net issuance is green, it creates a new obligation

$$\Delta m_G = \chi_G \ell^+ \mathbf{1}\{J = G\}.$$

The parameter χ_G maps green issuance into required adaptation or eligible spending.

The borrowing set also includes a minimum issuance-price condition. Following the long-bond sovereign default literature on debt dilution, the government cannot sell new debt at arbitrarily low prices. Let $\underline{q}$ denote this floor. Then positive net issuance must satisfy

$$\ell > 0 \implies q(y, x; g', B', m') \geq \underline{q}.$$

This is a market-access restriction, not a spread-trimming rule. If the sovereign is already carrying debt, the equilibrium secondary-market price of that debt may still fall below $\underline{q}$. The restriction only says that the government cannot use crisis prices to raise additional resources through new issuance. It prevents the numerical economy from generating borrowing booms immediately before default, a known problem in long-term-debt models and the reason Hatchondo et al. (2016) impose a minimum issuance price.

Green obligations turn promises into state variables. A sovereign can announce high future adaptation, but absent an inherited obligation state the future government re-optimizes. The state variable m changes future incentives through compliance costs and current-use requirements. Thus the model does not make green debt cheaper by assumption; it makes green issuance an imperfect implementation technology for future adaptation commitments. This is the quantitative analogue of Proposition 4.2: a label with no effect on m, feasibility, enforcement, or real allocations has no fundamental price effect.

The model also allows a current-use requirement. A fraction η_m of new green obligations must be matched by current adaptation. Let current unmet use be

$$u_m = \max\{\eta_m \Delta m_G - \max(i - m, 0), 0\}.$$

The next obligation stock is

$$m' = (1 - \delta_M)m + \Delta m_G + u_m.$$

The case $\eta_m = 0$ is the future-reporting-only model. Positive η_m captures tighter use-of-proceeds timing.

6.4 Compliance and Resource Costs

Inherited obligations and current-use requirements generate compliance costs when adaptation falls short:

$$\Psi(\bar{m} - i) = \psi_0 \max\{\bar{m} - i, 0\} + \frac{\psi_1}{2} \max\{\bar{m} - i, 0\}^2,$$

where $\bar{m}$ is the relevant obligation stock entering the current compliance check. The no-enforcement economy sets $\psi_0 = \psi_1 = 0$.

The repayment resource constraint is

$$c + i + \frac{\gamma_I}{2} i^2 + \Psi(\bar{m} - i) + \frac{\gamma_G}{2} \mathbf{1}\{J = G\}(\ell^+)^2 + (\kappa + \delta_B)B = \tilde{y}(y, x, g) + q(y, x; g', B', m')\ell.$$

The term γ_G captures eligible-project, certification, reporting, and monitoring capacity costs of scaling green issuance. The term γ_I captures adaptation implementation capacity: planning, procurement, execution, and monitoring costs. Neither term is an investor taste premium. For $\ell > 0$, the price in this budget constraint is also subject to the issuance floor above. For $\ell \leq 0$, the government is rolling down or reducing debt, so the floor is irrelevant.

6.5 Default and Repayment

If the sovereign defaults, it is excluded from markets and re-enters with probability ξ. Reentry resets debt and obligations to zero. The default value is

$$V^D(y,x,g) = u(c^D) + \beta \mathbb{E}\left[(1-\xi)V^D(y',x',g'_D) + \xi V(y',x',g'_D,0,0)\right],$$

where $g'_D = (1-\delta_G)g$. In repayment, the government chooses (g', B', J) subject to feasibility. The repayment action value is

$$W(s,a) = u(c(s,a)) + \beta \mathbb{E}[V(s')].$$

6.6 Taste Shocks and Pricing

The model uses EV1 taste shocks to smooth default and repayment choices. These are numerical regularization parameters, not the economics of the paper. With action-shock scale ρ_A,

$$V^{rep}(s) = \rho_A \log \sum_{a \in \mathcal{A}(s)} \exp\{W(s,a)/\rho_A\},$$

and action probabilities are logit probabilities. With default-shock scale ρ_D, default probabilities are

$$p_D(s) = \frac{\exp\{V^D(s)/\rho_D\}}{\exp\{V^D(s)/\rho_D\} + \exp\{V^{rep}(s)/\rho_D\}}.$$

Robustness checks vary ρ_A and ρ_D.

There is one fundamental price schedule:

$$q(y,x;g',B',m').$$

It is not indexed by the label except through the continuation state. The lenders' zero-profit condition is

$$q(y,x;g',B',m') = \frac{1}{1+r}\mathbb{E}_{y',x'|y,x}\left[(1-p_D(s'))\left(\kappa + \delta_B + (1-\delta_B)\bar{q}(s')\right)\right],$$

where $s' = (y',x',g',B',m')$ and $\bar{q}(s')$ is the action-probability-weighted continuation price. With $\kappa > r$, the default-free price can exceed one; the calibration therefore reports both face-value and market-value debt.

6.7 Model Objects

The model is used to construct four objects that correspond to the theory and empirical evidence.

First, the issuer-wide commitment gain compares the price or yield associated with green-access continuation policies to regular-only counterfactuals. This is the structural object that can be missed by a bond-level greenium. Second, the pass-through/dilution object compares issuer-wide debt valuation gains to the green bond's relative yield gain. This maps directly to Proposition 4.4. Third, additional adaptation compares adaptation under green access to adaptation under regular-only borrowing. This is the model counterpart of the public-allocation evidence. Fourth, welfare compares regimes using consumption-equivalent changes, because a lower spread is not sufficient for welfare improvement when obligations restrict future policy.

These objects are not unconditional observed greeniums. They are structural quantities disciplined by the empirical evidence: small bond-level greeniums, modest issuer-risk responses, heterogeneous additionality, and climate-state ambiguity.

6.8 Recursive Equilibrium

A recursive equilibrium consists of value functions $\{V, V^D, V^R\}$, default probabilities p_D, policy rules $(\pi_B, \pi_i, \pi_J, \pi_m)$, and a bond-price schedule q. Given q, the sovereign solves the repayment/default problem state by state. Default and repayment values satisfy

$$V(s) = \max\{V^D(y, x, g), V^R(s)\}$$

in the hard-choice benchmark, or the corresponding log-sum expression when taste shocks are active. Repayment policies solve the repayment problem subject to feasibility, the adaptation law of motion

$$g' = (1 - \delta_G)g + i,$$

the debt law of motion summarized by B', and the green-obligation law of motion

$$m' = (1 - \delta_M)m + \Delta m_G + u_m.$$

Competitive lenders price debt using the zero-profit condition

$$q(y, x; g', B', m') = \frac{1}{1+r}\mathbb{E}_{y',x'|y,x}\left[(1 - p_D(s'))\left(\kappa + \delta_B + (1 - \delta_B)\bar{q}(s')\right)\right],$$

where $s' = (y', x', g', B', m')$ and $\bar{q}(s')$ is the action-probability-weighted continuation price. Feasibility requires the repayment resource constraint to hold with $c \geq 0$.

This is the decentralized/no-commitment allocation relevant for the paper. Here, "decentralized" means competitive lender pricing together with a Markov-discretionary sovereign. It is not a full household–firm Walrasian economy.

7 Calibration

The calibration uses Brazil as the baseline quantitative economy. Brazil is useful because the project has country-specific income-process estimates, EM-DAT climate-disaster histories, sovereign-risk moments, green-bond allocation data, and inaugural sovereign sustainable-bond issuance information. The calibration has two roles. The first is to discipline the sovereign default environment: debt, spreads, default risk, and macro volatility. The second is to discipline the scale of green use and adaptation implied by the obligation technology. It does not target the observed bond-level greenium, because that object includes residual wedges outside the structural model.

7.1 Exogenous Processes

The output process follows the previous Brazil quantitative calibration. Log real GDP per capita is detrended and modeled as an AR(1), then converted to quarterly frequency and discretized with the Rouwenhorst method. Using the existing Brazil process keeps the sovereign default block comparable to the earlier project code and avoids re-estimating a central shock process while changing the model's green-bond block. The baseline uses

$$\rho_y = 0.980, \qquad \sigma_y = 0.019.$$

The climate process is estimated from the EM-DAT public event file. The baseline keeps Brazilian hydrological, meteorological, and climatological disasters from 2000 to 2024 and excludes geophysical events. A high-stress climate episode starts when adjusted damages exceed 0.10 percent of contemporaneous GDP; when damage is missing, an affected-population threshold is used and missing damage is not treated as zero. The resulting two-state process has a quarterly low-to-high probability of 0.0676 and a high-to-low probability of 0.152. The median quarterly GDP-equivalent loss in high-stress episodes is 0.000712. This parsimonious two-state process is intentionally conservative: it identifies disaster stress from public data but does not try to fit a rich climate-damage distribution with a short country time series.

7.2 External Parameters

The long-bond block follows the Chatterjee–Eyigungor convention used in the previous project code: quarterly risk-free rate $r = 0.01$, coupon $\kappa = 0.03$, and maturity parameter $\delta_B = 0.05$. These parameters fix the maturity and coupon structure of the debt instrument rather than being used to fit the green-bond facts. The default-free price is therefore

$$q^f = \frac{\kappa + \delta_B}{r + \delta_B} = 1.333.$$

The minimum issuance price is set to 70 percent of this default-free long-bond price,

$$\underline{q} = 0.70 q^f = 0.933.$$

This follows the calibration rule in Hatchondo et al. (2016), who use a minimum issuance price to prevent the model from letting governments issue large amounts of long-term debt at extremely low prices just before default. The restriction is applied only to positive net issuance. It does not cap the secondary-market price of outstanding debt and therefore does not mechanically censor simulated spreads. In the calibration output, $\underline{q}$ is recorded as an externally fixed market-access parameter and can be set to zero in validation runs that intentionally remove the Hatchondo-style borrowing floor.

Risk aversion is set to $\sigma = 2$, with robustness values. Reentry is $\xi = 1/20$, corresponding to a five-year average exclusion spell. Adaptation depreciates with a ten-year half-life, $\delta_G = 1 - 2^{-1/40}$. Green obligations decay over an eight-quarter reporting/allocation horizon, $\delta_M = 1/8$. These externally fixed values are chosen before the internal calibration because they define accounting horizons and standard sovereign-default primitives, not the strength of the green commitment channel.

The obligation intensity χ_G is measured from Brazil's parsed allocation annex. Narrow adaptation and resilience categories account for 3.69 percent of allocated sustainable-bond proceeds. A broader definition gives 7.49 percent and is used for robustness.

7.3 Internal Parameters and Targets

The internally calibrated vector is

$$(\beta, \phi_0, \phi_1, \alpha_G, \psi_0, \psi_1, \eta_m, \gamma_G, \gamma_I).$$

The first three parameters discipline the sovereign default core. The parameter α_G governs adaptation effectiveness. The parameters ψ_0 and ψ_1 govern enforcement of unmet obligations. The parameter η_m governs current-use timing. The parameter γ_G governs green-label capacity costs. The parameter γ_I governs adaptation implementation costs. Joint calibration is important because the same observed green share can be produced by high commitment value and high capacity cost, or by weak commitment value and low capacity cost. The sovereign-risk moments discipline the first margin; the green-use moments discipline the second.

The sovereign-risk targets are mean spreads, market-access debt-to-GDP, annual default frequency, spread volatility, and log consumption volatility. The green-use targets are the green share among positive net issuance states, adaptation investment per unit of green issuance, and a low-weight adaptation-flow-to-GDP target constructed from Brazil's parsed adaptation allocation divided by World Bank nominal GDP.

The observed greenium is an untargeted validation moment. This is necessary because the model does not contain bond-specific liquidity, scarcity, clientele, or index wedges. A model that targeted the observed greenium directly would force a structural commitment technology to explain residual bond-market wedges that the theory explicitly separates from issuer-wide value.

7.4 Benchmark Calibration

The benchmark calibration estimates γ_I jointly with the other internal parameters on the $9 \times 2 \times 5 \times 51 \times 11$ grid. It uses the Fortran probability-window solver, 28 parallel Julia workers, single-threaded Fortran solves, 44 differential-evolution population members, eight global generations, and four local refinement passes. The selected vector passes the strict convergence screen: the value-function error is 1.24×10^{-6}, the price-recursion error is 9.88×10^{-6}, the tolerance is 10^{-5}, and the solution converges in 426 iterations out of an 800-iteration cap. The scalar loss is 7.28.

The calibrated vector is

$$\beta = 0.968, \quad \phi_0 = -0.433, \quad \phi_1 = 0.486, \quad \alpha_G = 2.500,$$
$$\psi_0 = 0.098, \quad \psi_1 = 12.000, \quad \eta_m = 0.118, \quad \gamma_G = 16.029, \quad \gamma_I = 208.313.$$

The benchmark fits the sovereign default frequency, mean spreads, green issuance, and adaptation intensity reasonably well. It implies an annual default frequency of 2.50 percent, a mean spread of 279 basis points, a green issuance share of 13.7 percent, and an adaptation share of green issuance of 3.5 percent. It underfits two moments: market-access debt is 0.77 rather than 0.85, and CE-style spread volatility is 214 basis points rather than 443 basis points. These misses are reported rather than hidden because they matter for interpretation. The benchmark is strong on the central default and green-use margins but conservative about crisis-state spread volatility.

Table 6: Benchmark calibrated parameters

Parameter	Interpretation	Value
β	quarterly discount factor	0.9676
ϕ_0	output cost intercept in default	-0.4329
ϕ_1	output cost slope in default	0.4863
α_G	adaptation effectiveness	2.5000
ψ_0	obligation enforcement level	0.0975
ψ_1	obligation enforcement curvature	12.0000
η_m	current-use timing share	0.1181
γ_G	green-issuance capacity cost	16.0288
γ_I	adaptation implementation cost	208.3134

Notes: The vector is from the production run dated 2026-05-30. It is used as the benchmark for the quantitative results. The vector is reported because the green-bond mechanism is jointly disciplined by default-risk moments, green issuance, adaptation intensity, and strict numerical convergence.

The spread-volatility gap is economically informative rather than a numerical failure. Without the minimum issuance-price restriction, the long-bond model can generate large spread volatility through extremely low-price pre-default borrowing. That tail is not a credible mechanism for the paper's green-bond question. With the restriction in place, the benchmark asks the model to generate spread volatility from genuine default-risk movements while also matching default frequency, debt, green issuance, and adaptation. This leaves the model deliberately conservative about the role of green debt in volatile crisis states, and the policy counterfactuals should be read with that limitation in mind.

8 Quantitative Results

The quantitative results put magnitudes on the paper's central distinction. The empirical greenium measures the price of one labelled bond relative to an imperfect conventional counterfactual. The model asks a different question: how much value can a credible green-obligation technology create for the sovereign issuer as a whole, how is that value shared across pari-passu claims, and when does the obligation become too rigid to be welfare improving?

This distinction is important because the empirical evidence deliberately leaves the paper with an identification problem. Small bond-level greeniums can mean that investors do not value the label, that programs are not additional, or that the fundamental value of the covenant has passed through to conventional bonds and CDS. Reduced-form bond prices alone cannot separate those cases. The role of the quantitative model is to discipline the mechanism: it imposes a single fundamental price schedule for all pari-passu debt, allows green obligations to affect prices only through future allocations and default risk, and then asks whether the resulting issuer-wide gains can be economically meaningful even when the residual bond-level greenium is small.

The section is organized around three questions. First, does the calibrated economy remain a credible sovereign-default economy once the green-bond block is added? Second, does access to green borrowing create value because it is a state-contingent commitment option, or because the model has mechanically made green debt cheap? Third, where does the value appear: in the labelled bond, in the whole debt stock, in additional adaptation, or in welfare? These questions correspond to the calibration fit, the regime comparisons, and the state-level decomposition below.

8.1 Fit and Quantitative Discipline

Table 7 reports the calibrated moments. This table matters because the model should not obtain green-bond gains by abandoning the sovereign-default discipline that makes the exercise useful. The benchmark is therefore a conventional sovereign-default economy before it is a green-bond model: it is disciplined by mean spreads, debt, default frequency, consumption volatility, and spread volatility. Only after those margins are present does the calibration ask the green block to match the share of green issuance among positive net issuance states and adaptation investment per unit of green issuance.

The observed bond-level greenium is deliberately left as an untargeted validation object. This is not a weakness of the calibration; it follows from the theory. The structural model contains no bond-specific liquidity, scarcity, index, tax, or clientele wedge. Forcing the model to match the observed greenium would incorrectly ask a fundamental commitment channel to explain residual market-segmentation components that the decomposition assigns to Λ_{bct} and $\mathcal{R}_{bct}$. The correct test is instead whether a model disciplined by sovereign risk and green-use moments produces plausible issuer-wide commitment gains while remaining consistent with small residual greeniums.

The benchmark matches mean spreads, default frequency, green issuance, and adaptation intensity reasonably well. Its main limitation is spread volatility. The model produces less CE-style spread volatility than the calibration target once the Hatchondo-style minimum issuance-price restriction rules out crisis borrowing at extremely low prices. That miss is reported explicitly because it changes how the results should be read. The commitment gains below do not come from an explosive crisis tail in which the sovereign issues large amounts of debt immediately before default. They come from ordinary states in which a future allocation constraint changes repayment incentives.

Table 7: Calibration fit: sovereign-risk moments and green-use moments

Moment	Model	Target	Deviation	Loss share
Mean spread, bps	279.46	250.00	29.46	0.35
Debt/GDP, market access	0.767	0.850	-0.083	1.39
Annual default frequency	0.025	0.026	-0.001	0.04
CE spread volatility, bps	214.08	443.00	-228.92	1.75
Log consumption volatility	0.140	0.068	0.071	0.64
Green issuance share	0.137	0.150	-0.013	0.28
Adaptation share of green issuance	0.035	0.037	-0.002	0.01
Adaptation investment/GDP	0.00072	0.000004	0.00072	2.57
Share CE spread above 1000 bps	0.004	0.000	0.004	0.26
Share CE spread above 2000 bps	0.000	0.000	0.000	0.00

Notes: The table reports the benchmark using the $9 \times 2 \times 5 \times 51 \times 11$ grid, the Fortran probability-window solver, and the Hatchondo-style minimum issuance-price restriction $\underline{q} = 0.70q^f = 0.933$. The scalar loss is 7.28. The calibration run uses quarterly model periods, a 10,000-period simulation with 2,000 burn-in periods, and 28 Julia workers calling single-threaded Fortran solves. The vector passes the strict convergence screen with value-function error 1.24×10^{-6}, price-recursion error 9.88×10^{-6}, tolerance 10^{-5}, and 426 iterations out of an 800-iteration cap. The main miss is spread volatility, so the counterfactuals should be read as conservative about volatile crisis states.

8.2 Regime Results

The first substantive experiment is the regime comparison in Table 8. The comparison is designed to separate access, compulsion, and empty labels. The regular-only economy removes the commitment technology. The green-access economy lets the government choose when to use it. The mandatory-green economy forces the government to attach the obligation to all positive net issuance. The no-enforcement economy keeps the label but removes the compliance force that can change future allocations.

Relative to regular-only borrowing, green access lowers mean spreads from 398 to 270 basis points, lowers the annual default frequency from 2.87 to 2.48 percent, leaves average debt essentially unchanged, and raises initial-state welfare by 1.29 percent of consumption. These are issuer-wide effects, not bond-specific label premia. They arise because, in some states, green issuance commits the government to future adaptation in a way that lenders value when pricing the whole sovereign debt stock. The no-enforcement rows coincide with regular-only borrowing, confirming the theoretical restriction that labels have no fundamental value unless they change real allocations or enforcement incentives.

The mandatory-green row is equally important because it rules out a naive interpretation of the model. If green debt were simply cheaper debt, making all issuance green should be desirable. It is not. Mandatory green issuance sharply lowers spreads and default risk, but it also collapses debt capacity and reduces welfare. The reason is economic rather than numerical: forcing every positive net issuance into the green-obligation technology removes fiscal flexibility precisely when flexibility is valuable. This is the model's first policy lesson. Green bonds are valuable as access to a commitment option, not as a mandate that all sovereign borrowing be green.

Table 8: Green access creates option value; mandatory green issuance destroys flexibility

Regime	CE gain	Spread	Spread vol.	Default	Debt/GDP	Green share	Adapt./green
Regular only	0.00	397.79	1067.42	2.87	0.776	0.000	0.000
Green access	1.29	269.69	216.75	2.48	0.774	0.140	0.035
Mandatory green	-3.36	106.89	460.60	0.57	0.164	1.000	0.002
No enforcement: regular	0.00	397.79	1067.42	2.87	0.776	0.000	0.000
No enforcement: duplicate labels	0.00	397.79	1067.42	2.87	0.776	0.000	0.000

Notes: CE gain is the initial-state consumption-equivalent gain, in percent, relative to the regular-only regime. Spreads and spread volatility are in basis points. Default frequency is annual, in percent. All rows are solved with the repaired Fortran probability-window backend, 28 OpenMP threads, tolerance 10^{-5}, and the benchmark parameter vector. The green-access row converges in 426 iterations. The regular-only and no-enforcement rows are solved on a collapsed one-point obligation grid because green liabilities cannot be created when only conventional debt is available.

8.3 Where Commitment Value Appears

The state-level results explain why the regime gain is consistent with a small bond-level greenium. The green obligation is not valuable in every state. It is valuable before the government has accumulated costly obligations and when additional adaptation meaningfully improves repayment capacity. It can be costly after obligations have been inherited, especially when the state calls for fiscal flexibility rather than more earmarked spending.

Table 9 reports the green-versus-regular yield advantage implied by the solved state grid. Across all states, the mean commitment gain is 6 basis points and the median is zero. This average is small because the grid includes states with inherited green obligations, and those obligations can be costly ex post. Starting from no inherited obligations, which is the clean access margin, the mean commitment gain is 18 basis points and is positive in 84 percent of states.

Table 9: Commitment value is concentrated at the no-inherited-obligation access margin

State sample	N	Mean	Median	P90	Share > 0
All grid states	50,490	6.05	0.00	15.88	0.41
No inherited obligations	4,590	17.90	2.12	40.27	0.84

Notes: Commitment value is the green-minus-regular yield advantage in basis points, computed from the Fortran solution at each state. The no-inherited obligation sample fixes $m = 0$, so it is the cleanest state-grid analogue of a sovereign deciding whether access to green borrowing creates commitment value before old green obligations are present. The all-state average is smaller because inherited obligations can be costly ex post.

The difference between the all-state and no-inherited-obligation rows is central. Green access creates option value before obligations are accumulated. Once obligations are inherited, they are state variables that discipline future policy but can also reduce fiscal flexibility. A serious model of sovereign green bonds should have both forces. Otherwise it would mechanically treat green debt as cheap debt and would miss the main policy tradeoff.

8.4 Pass-Through and Additionality

Table 10 connects the model to the empirical decomposition. This is the key table for interpreting the empirical greenium. If the green covenant improves repayment capacity, the gain is not confined to the labelled bond. It raises the value of conventional claims as well because all pari-passu creditors are exposed to the same default event. The observed greenium can therefore be small either because there is little fundamental value or because the fundamental value has been capitalized into the rest of the sovereign curve.

In the no-inherited-obligation sample, green access raises issuer-wide value, raises the value of conventional claims, and lowers default risk relative to regular-only borrowing. That is the pass-through mechanism. The bond-level green relative yield gain is only one part of the issuer-wide effect, and it can understate the welfare-relevant value of the covenant.

Table 10: Issuer-wide pass-through is larger than the labelled-bond residual object

State sample	Green yield gain	Issuer value gain	Conventional price gain	Δp_D	Mean add. invest.	Share add. > 0
All grid states	6.05	-0.232	-0.457	33.61	0.000134	0.45
No inherited obligations	17.90	0.051	0.079	-4.10	-0.000003	0.43
Positive debt, no obligations	18.19	0.052	0.080	-4.18	-0.000003	0.42
Positive-additionality states	–	–	–	–	0.000306	1.00

Notes: Yield gains and default-probability responses are in basis points and percentage points, respectively. Issuer value gain is $(q^{access} - q^{regular})B$. The all-state row averages over inherited green-obligation states; the no-inherited-obligation rows isolate the ex-ante access margin. Additionality is the difference between expected adaptation investment under green access and regular-only borrowing. The table separates the issuer-wide value channel from the bond-level green yield gain emphasized in reduced-form greenium estimates.

The same table also reports additionality. The benchmark produces positive additional adaptation in a substantial share of states, but the average additional investment at the clean $m = 0$ access margin is close to zero. This is a useful discipline on the paper's interpretation. The model does not say that every green bond mechanically creates large new adaptation. It says that commitment value requires the combination of credible obligations, fiscal capacity, and a state in which adaptation changes repayment risk. This is why the empirical allocation evidence is not ancillary. Without evidence on use-of-proceeds timing, adaptation content, and reporting credibility, a measured greenium cannot tell us whether the label financed marginal climate investment or simply reclassified spending that would have occurred anyway.

8.5 Climate-State Heterogeneity

The climate-state exercise asks whether the model delivers a simple prediction that more climate exposure should always imply a larger greenium. It does not, and that is an important result. Climate vulnerability raises the potential value of adaptation, but it can also raise financing needs, default risk, and the cost of making a credible promise. The same state that makes adaptation socially valuable can make the green obligation fiscally fragile.

Table 11 shows that, under the two-state EM-DAT climate process, the mechanism is nearly flat across low- and high-stress climate states. This is not evidence that climate risk is irrelevant. It says that the calibrated climate process is too coarse to generate strong climate-state heterogeneity by itself. The stronger prediction is conditional: climate vulnerability should amplify commitment value only when adaptation materially changes repayment capacity and the sovereign has enough fiscal space for the obligation to remain credible.

This result also clarifies the empirical evidence. A weak unconditional relationship between observed greeniums and climate vulnerability need not contradict the model. The observed

Table 11: The climate-state gradient is weak in the two-state EM-DAT calibration

Climate state	Green issuance prob.	Commitment gain	Δp_D	State-grid welfare gain
Low climate stress	0.407	6.01	33.60	-1.19
High climate stress	0.406	6.08	33.62	-1.19

Notes: Values are unweighted averages over the state grid by climate state. Commitment gains are in basis points; Δp_D is in percentage points; welfare gains are consumption-equivalent percent relative to the regular-only state value. These are mechanism-map objects, not simulated ergodic moments. The result says that the benchmark climate process is too coarse to generate a strong monotone climate-gradient in commitment value by itself.

residual greenium mixes commitment value, pass-through, supply, liquidity, and demand. Moreover, high climate vulnerability can raise both the value of adaptation and fiscal stress. The model therefore predicts state-contingent, not monotone, pricing effects. This is why the paper treats climate-risk heterogeneity as a mechanism test rather than as a one-dimensional prediction that vulnerable countries should always have larger residual greeniums.

8.6 Interpretation

The quantitative results deliver three messages. First, small observed greeniums can coexist with issuer-wide commitment value because default-risk improvements pass through to conventional claims. This is the quantitative counterpart of the paper's decomposition. Second, commitment value is an option value: green access improves welfare in the benchmark, while mandatory green issuance lowers welfare. The instrument is valuable because the sovereign can use it selectively. Third, additionality is not automatic. The model's positive welfare result depends on the sovereign using the obligation technology in states where it is credible and valuable, not on a mechanical green label price.

These conclusions also define what the quantitative exercise does not claim. It does not claim that every sovereign green bond creates new adaptation. It does not claim that the observed greenium is the welfare gain. And it does not claim that green mandates are desirable. The model's contribution is narrower and more useful: it shows that a credible, state-contingent green-obligation technology can create issuer-wide value even when the bond-level residual greenium is small, but that this value disappears without enforcement and can reverse when the obligation is imposed too rigidly.

9 Welfare and Policy

The welfare analysis separates two ideas that are often conflated in discussions of green finance. A green bond can lower measured borrowing costs because it changes default risk, but lower spreads are not sufficient for higher welfare. The covenant also restricts fiscal choices. A policy that makes debt safer by preventing the government from borrowing or reallocating resources in bad states can reduce default risk and still make the sovereign worse off. The relevant welfare question is therefore whether the commitment value of the obligation is large enough to compensate for the loss of flexibility.

This section uses the model as a policy laboratory. The exercises should not be read as institutional blueprints for any one country. They are designed to isolate margins that policymakers actually control: whether green borrowing is optional or mandatory, whether use-of-proceeds promises are enforced, whether adaptation is subsidized, and whether external guarantees lower rollover risk. The purpose is to translate the paper's decomposition into policy language. A bond-level greenium is a price residual; welfare depends on default risk, fiscal flexibility, additional investment, and the credibility of the commitment technology.

9.1 Regime Comparisons

The model compares regular-only borrowing, green access, mandatory green issuance, and no-enforcement labels. Each regime removes one possible interpretation of the results. Regular-only borrowing is the benchmark without green commitment. Green access allows the sovereign to choose conventional or green issuance state by state. Mandatory green issuance asks whether the instrument remains valuable when it is no longer optional. The no-enforcement economy sets compliance costs to zero and removes any direct green price wedge, so it tests whether the model is creating value from enforcement and real allocations rather than from the label itself.

Table 8 reports the main welfare comparison. Green access improves welfare relative to regular-only borrowing. The gain is not a label premium: when enforcement is removed, the economy collapses back to the regular-only allocation. The gain comes from state-contingent commitment. The sovereign uses green borrowing when the obligation improves future repayment incentives enough to justify the constraint, while preserving conventional borrowing when the obligation would be too costly. This is why welfare and spreads need not move one-for-one. The spread is a market price of repayment risk; welfare also values consumption smoothing, debt capacity, and the option to avoid an obligation in states where it is not useful.

Mandatory green issuance has the opposite welfare implication. It lowers spreads and default frequency, but it does so by sharply reducing debt capacity and fiscal flexibility. This is why the model does not support a green-bond mandate. The policy-relevant object is access to a commitment technology, not compulsory use of that technology in every financing state.

9.2 Policy Complements

Green access may be weakest where climate need is greatest. High-debt or highly vulnerable sovereigns can have valuable adaptation opportunities but limited fiscal credibility. This creates a policy problem that the bond label alone cannot solve. If investors do not believe the obligation will translate into repayment capacity, or if the sovereign cannot finance the complementary adaptation investment, the commitment technology may remain unused or become too costly.

The model therefore evaluates three reduced-form complements. Concessional green finance raises the issuance price of positive net green borrowing. An externally financed adaptation matching grant lowers the government's direct adaptation cost. A partial guarantee adds a recovery payoff to lenders in default. These are not complete institutional designs, but they isolate three policy margins: cheaper green issuance, cheaper adaptation, and lower rollover/default-risk premia.

Table 12: Moderate policy complements improve the green-access benchmark only modestly

Policy	CE gain	Spread	Spread vol.	Default	Green share
Concessional green finance, 25 bp price wedge	1.291	271.14	218.61	2.38	0.137
Concessional green finance, 50 bp price wedge	1.291	269.86	217.16	2.35	0.139
Adaptation matching grant, 25 percent	1.294	269.52	218.44	2.28	0.140
Partial guarantee, 25 percent recovery	1.294	272.28	217.62	2.35	0.141

Notes: CE gains are initial-state consumption-equivalent gains, in percent, relative to regular-only borrowing. Spreads and spread volatility are in basis points; default frequency is annual, in percent. All reported rows pass the strict Fortran convergence screen. The 50 percent grant and 50 percent guarantee stress tests are implemented in the Fortran kernel but are excluded from the main rows because they did not satisfy the strict price-recursion tolerance within 2,500 iterations. The main policy lesson is that complements can help, but the dominant gain in the benchmark comes from access to the state-contingent commitment option.

The moderate policy complements in Table 12 move the benchmark only modestly. A 25 percent adaptation matching grant and a 25 percent partial guarantee both raise initial-state welfare slightly relative to the green-access benchmark and reduce default risk relative to regular-only borrowing. The effects are not large enough to overturn the central result: the primary welfare gain comes from green access as a commitment option. This is informative. In the calibrated Brazil economy, modest external support improves the margin but does not substitute for credible domestic use-of-proceeds enforcement. Larger 50 percent grant and guarantee stress tests are implemented in the Fortran kernel, but they are excluded from the main table because the price recursion did not satisfy the strict convergence tolerance within 2,500 iterations.

9.3 Policy Implications

The policy implication is conditional. Sovereign green bonds can improve welfare when they create credible, additional commitments that strengthen repayment capacity. They are not a general-purpose subsidy to sovereign borrowing. Programs dominated by retrospective accounting, weak enforcement, or non-additional spending should not generate fundamental value in the model. Conversely, highly vulnerable sovereigns may need complementary grants, guarantees, concessional finance, or debt relief before green obligations become credible and additional.

The broader implication is that green-bond policy should be evaluated as part of a fiscal-climate financing package. The bond label alone is not enough. What matters is whether the instrument changes the sovereign's intertemporal incentives in a state where adaptation affects repayment risk and where the obligation does not destroy valuable fiscal flexibility. This also clarifies the empirical contribution of the paper: public allocation reports, impact reports, timing of expenditures, and issuer-risk responses are not secondary details. They are the information needed to decide whether a sovereign green-bond program is likely to operate as a commitment device or only as a labelled financing instrument.

10 Conclusion

The sovereign greenium is small because it is not the right structural object. Sovereign green bonds are normally pari-passu claims on the same issuer as conventional debt. If a green covenant improves repayment capacity, the fundamental gain should not remain trapped inside the labelled bond. It should pass through to the sovereign's whole debt stock. The relative price of the green bond therefore measures a residual object: special demand, liquidity, scarcity, index eligibility, tax treatment, benchmark status, and any imperfect trace of the issuer-wide commitment component.

This distinction changes how sovereign green bonds should be evaluated. The relevant question is not whether the average labelled bond trades many basis points rich to a matched conventional bond. The relevant question is whether the program changes future fiscal choices in a way that markets regard as credible and repayment-relevant. A transparent framework can matter, but transparency alone is not additionality. A second-party opinion can certify eligible categories, but certification alone does not guarantee new investment. A green bond creates fundamental value only when the covenant changes the path of spending, adaptation, transition investment, or fiscal-risk management.

The empirical evidence points in that direction. Bond-level greeniums are small and sensitive to whether one uses yields, asset-swap spreads, or z-spreads. Event-study responses in CDS and matched conventional bonds are modest, which is inconsistent with a large unconditional label-premium story. At the same time, public allocation and impact reports show substantial heterogeneity in adaptation content, reporting quality, and accounting timing. Some programs look closer to forward-looking commitment devices; others look closer to transparent refinancing or expenditure classification. That heterogeneity is not a nuisance around the main result. It is central to understanding why average greeniums are small.

The model gives this interpretation a quantitative structure. Green debt is not assumed to be cheaper. The label affects prices only through the continuation state created by green obligations, compliance incentives, adaptation investment, and default risk. In the Brazil benchmark, access to the green-obligation technology lowers spreads and default risk and raises welfare relative to regular-only borrowing. But mandatory green issuance is welfare reducing because it removes fiscal flexibility. The model therefore does not support the view that all sovereign debt should be green. It supports a narrower claim: green bonds can be useful commitment options when credibility, additionality, and fiscal capacity line up.

The policy implication is correspondingly conditional. For countries with credible institutions and projects that change future fiscal risk, green bonds may lower financing costs in a way that is mostly issuer-wide rather than bond-specific. For countries with weak credibility, high debt, or severe climate exposure, the same obligation may need complementary grants, guarantees, concessional finance, or debt relief to become both credible and welfare improving. This is why the policy discussion cannot stop at the observed greenium. A small greenium may mean that the bond-level wedge is small, not that the program has no commitment value.

The paper also leaves a measurement agenda. Better sovereign green-bond data should separate new spending from refinancing, mitigation from adaptation, current from retrospective allocation, and labelled-bond pricing from issuer-wide risk repricing. Those distinctions are necessary for testing whether green bonds are mainly investor-demand assets, disclosure devices, or genuine fiscal commitment technologies. The evidence here is most consistent with the commitment interpretation only under specific conditions. That conditionality is the main economic message: sovereign green bonds are not cheap debt by construction, but they can create value when they credibly change the sovereign's future policy problem.

A Theory Appendix

The main text includes the economic figures that correspond to the formal claims. Figure 3 illustrates the threshold comparative statics. Figure 4 illustrates the underinvestment margin. Figure 5 summarizes the credibility/additionality condition behind Proposition 4.2. Figure 2 illustrates the pass-through logic in Proposition 4.4. This appendix records the short proofs so that the main text can emphasize the economic interpretation.

A.1 Proof of Lemma 4.1

Let $X_j(h)$ be the state-contingent cash flow of bond $j \in \{a, b\}$ in future history h. The hypotheses imply $X_a(h) = X_b(h)$ for every history. With competitive pricing under a common stochastic discount factor $M(h)$,

$$q_j(s) = \mathbb{E}_s[M(h)X_j(h)].$$

Identical state-contingent cash flows have identical present values. Hence $q_a(s) = q_b(s)$.

A.2 Threshold Comparative Statics

Under Assumption 1, the default threshold satisfies

$$\Delta(\bar{\varepsilon}(B,i); B, i) = 0.$$

The implicit function theorem gives

$$\bar{\varepsilon}_B = -\frac{\Delta_B}{\Delta_\varepsilon} > 0, \qquad \bar{\varepsilon}_i = -\frac{\Delta_i}{\Delta_\varepsilon} < 0.$$

The price derivative is

$$q_i(B,i) = -\frac{1}{1+r} f(\bar{\varepsilon}(B,i))\bar{\varepsilon}_i(B,i) > 0.$$

A.3 Proof of Proposition 4.2

If the green label does not change real policy, enforcement, resources, recovery, liquidity, taxes, investor constraints, or the feasible set, the green and regular policy problems are identical. Thus $i^G = i^R$ and $q(B, i^G) = q(B, i^R)$. The issuer-wide fundamental commitment value is zero. If $i^G > i^R$, then $q_i > 0$ implies $q(B, i^G) > q(B, i^R)$.

A.4 Proof of Proposition 4.4

Let s^R and s^G denote continuation states under regular and green contracts. For any pari-passu claim k, the valuation change is

$$\Delta q_k = \mathbb{E}[M(h)(X_k(h; s^G) - X_k(h; s^R))].$$

If the covenant lowers the probability of the common default event, then the state-contingent payoff distribution changes for every claim exposed to that event. The size of Δq_k can vary with maturity, coupon, and recovery, but the fundamental effect is not exclusive to the green claim. Therefore a green-conventional relative yield identifies the full issuer-wide value only if the conventional comparison claim receives no pass-through, which is a knife-edge case inconsistent with common default exposure.

B Empirical Appendix

This appendix collects construction details, robustness checks, and mechanism evidence that are useful for interpreting the main empirical facts but too detailed for the body of the paper.

B.1 Bond Panels and Matching

The daily ISIN panel uses the FactSet workbook in the raw-data folder. The duration and price sheets share the date axis from the yield sheet. The monthly individual-bond panels use the Europe and Americas sovereign-bond files. Issuer country is inferred from sovereign issuer names rather than trading-market fields, which prevents dollar sovereign bonds issued by Latin American countries from being matched to U.S. Treasuries. Green-conventional pairs are formed within issuer and currency with maturity-gap calipers; the preferred monthly sample uses a maximum maturity gap of two years.

Table 13: Benchmark-tenor residual greenium estimates

Measure	Obs.	Green ISINs	Countries	Mean bps	Median bps
ASW	765	23	8	-12.82	0.36
Yield	1223	35	11	0.49	0.02
z-spread	765	23	8	-16.76	0.65

Notes: Monthly green bonds are matched to the nearest same-country benchmark tenor. Positive values mean the benchmark yield or spread exceeds the green-bond yield or spread. This is a broader curve comparison, not the preferred individual-bond counterfactual.

Table 13 reports benchmark-tenor residuals. These are useful supporting estimates but not the preferred estimator because benchmark bonds differ from ordinary bonds in liquidity, repo specialness, curve status, and investor use.

Table 14 adds liquidity, climate, rating, and outlook controls to the matched-bond panel. The specifications are descriptive because the country dimension is limited and several controls are highly persistent at the issuer level.

Table 14: Liquidity, climate, and sovereign-risk controls in individual matches

Outcome	Spec.	Obs.	Bid-ask	Climate	Rating	Outlook
Yield	Liquidity	1046	0.343			
Yield	Climate	1140		-0.013		
Yield	Liq.+climate	1046	1.745	-0.017		
Yield	Country rating	1132			0.007	
Yield	Liq.+clim.+rating	1038	1.779	-0.018	-0.010	
Yield	Outlook	1104				0.118
ASW	Liquidity	1051	-130.446			
ASW	Climate	1134		-0.437		
ASW	Liq.+climate	1051	-127.867	-0.031		
ASW	Country rating	1126			0.172	
ASW	Liq.+clim.+rating	1043	-129.483	-0.028	-0.095	
ASW	Outlook	1098				-7.066
z-spread	Liquidity	1051	-71.601			
z-spread	Climate	1134		-0.119		
z-spread	Liq.+climate	1051	-79.221	0.091		
z-spread	Country rating	1126			-0.174	
z-spread	Liq.+clim.+rating	1043	-79.681	0.061	-0.287	
z-spread	Outlook	1098				-0.949

Notes: Matched monthly observations with maturity gap no larger than two years. Outcomes are pricing residuals defined so positive values are green richness. Bid-ask is the green bond bid-ask gap minus the matched conventional bond bid-ask gap. Climate is the latest FactSet LSEG SSRM climate-risk score. Rating and outlook are issuer-country controls built from `sovereign_ratings_history.csv` and, where missing, the benchmark-bond rating panel. Lower numerical ratings are safer. These are descriptive controls, not causal estimates.

Table 15: Robustness of individual-bond matched greenium estimates

Sample	Measure	Obs.	Green ISINs	Mean bps	Median bps
gap<=2y	Yield	1140	29	-0.06	0.01
gap<=2y	ASW	1134	28	2.23	0.54
gap<=2y	z-spread	1134	28	1.10	0.87
gap<=1y	Yield	987	26	-0.08	0.01
gap<=1y	ASW	1006	26	1.61	0.45
gap<=1y	z-spread	1006	26	0.55	0.74
Europe	Yield	795	15	-0.11	0.01
Europe	ASW	847	16	0.11	0.55
Europe	z-spread	847	16	0.77	0.87
Americas	Yield	345	14	0.04	0.00
Americas	ASW	287	12	8.47	0.45
Americas	z-spread	287	12	2.08	0.80
winsor 1/99	Yield	1140	29	-0.07	0.01
winsor 1/99	ASW	1134	28	2.72	0.54
winsor 1/99	z-spread	1134	28	1.44	0.87

Notes: All rows use same-country, same-currency individual conventional-bond matches. The regional rows require a maturity gap no larger than two years. Winsorization is applied to the baseline two-year-gap residuals at the 1st and 99th percentiles.

Table 15 reports maturity-caliper, winsorization, and regional robustness checks. The Americas have larger spread residuals, but the sample is smaller and reflects an external-debt market segment that differs from the euro-area sample.

Table 16: Leave-one-country-out matched greenium means

Dropped country	Countries left	Yield	ASW	z-spread
Brazil	11	-0.06	3.60	1.81
Canada	11	-0.06	2.28	1.11
Colombia	11	-0.07	2.64	1.16
Dominican Republic	11	-0.06	2.31	1.13
France	11	0.02	2.76	1.17
Germany	11	-0.10	2.73	1.07
Ireland	11	-0.07	2.28	0.82
Italy	11	-0.06	2.38	1.08
Lithuania	11	-0.07	2.92	1.62
Mexico	11	-0.06	2.27	1.12
Peru	11	-0.08	-0.61	0.37
Uruguay	11	-0.08	1.46	0.74

Notes: Means are recomputed after dropping each issuer country from the baseline individual-bond matched sample with maturity gap no larger than two years.

Table 16 shows that the average matched greenium is not driven by a single issuer in a way that changes the paper's main conclusion.

B.2 Spread Measures

Yield differences, asset-swap spread differences, and z-spread differences measure different objects. Yield differences include rate-curve and coupon effects. ASW and z-spread residuals are closer to relative spread pricing, but they still include liquidity, benchmark, and modeling components. The paper therefore treats consistency across measures as informative and disagreement across measures as evidence that the bond-level greenium is not a clean structural object.

B.3 Public Events and Issuer-Wide Risk

Table 17 reports the original CDS event-study design using the local 5-year CDS files. Framework and second-party-opinion events are followed by small CDS declines in some windows, but the estimates are statistically weak and not stable across outcomes.

Table 18 adds daily changes in the MOVE index to absorb global bond-market volatility. Once this control is included, framework/SPO coefficients become small and statistically weak.

Table 19 reports the earlier bond-level event-study specifications around public credibility events.

Table 20 reports the event-level pass-through estimates underlying Figure 8. Standard errors are shown only when there are at least three usable event-level paired changes. This restriction

Table 17: Issuer-level 5-year CDS event studies

Event group	Outcome	Window	Events	Post	S.E.
All events	CDS level	30	21	-0.072	0.494
All events	CDS level	90	21	-0.612	0.595
All events	Daily CDS change	30	21	-0.018	0.022
All events	Daily CDS change	90	21	0.005	0.014
Framework/SPO	CDS level	30	10	-0.251	0.236
Framework/SPO	CDS level	90	10	-0.305	0.721
Framework/SPO	Daily CDS change	30	10	-0.051	0.034
Framework/SPO	Daily CDS change	90	10	0.014	0.016
Reporting	CDS level	30	10	0.126	1.051
Reporting	CDS level	90	10	-1.071	0.989
Reporting	Daily CDS change	30	10	0.013	0.022
Reporting	Daily CDS change	90	10	-0.009	0.024

Notes: The CDS panel uses local 5-year sovereign CDS series from WorldGovernmentBonds and Investing.com. Regressions include event fixed effects and cluster by event. Coefficients are post-event changes in basis points; the daily-change outcome uses first differences of CDS spreads.

Table 18: MOVE-adjusted CDS change responses

Event group	Window	Events	Post coef.	SE
Framework/SPO	30	9	-0.041	0.031
Framework/SPO	60	9	0.017	0.023
Reporting	30	11	0.066	0.032
Reporting	60	11	0.023	0.023
All public events	30	24	0.015	0.019
All public events	60	24	0.027	0.015

Notes: Outcome is the daily change in 5-year CDS spreads. Regressions include event fixed effects and control for the daily change in the ICE BofA MOVE index. Coefficients are descriptive short-window post-event changes in CDS basis points.

Table 19: Bond-level greenium event studies around public credibility events

Event group	Outcome	Window	Events	Post	S.E.
All events	Curve residual	30	13	-0.07	0.15
All events	Curve residual	90	13	-0.07	0.31
All events	Matched pair	30	7	-0.42	0.40
All events	Matched pair	90	8	-0.27	0.30
Framework/SPO	Curve residual	30	5	-0.13	0.31
Framework/SPO	Curve residual	90	5	-0.04	0.48
Framework/SPO	Matched pair	30	3	-0.46	0.79
Framework/SPO	Matched pair	90	4	0.18	0.13
Reporting	Curve residual	30	8	-0.02	0.19
Reporting	Curve residual	90	8	-0.09	0.44
Reporting	Matched pair	30	3	-0.38	0.30
Reporting	Matched pair	90	3	-0.88*	0.53

Notes: Each estimate stacks event windows around audited framework, second-party-opinion, allocation-reporting, and impact-reporting dates. Regressions include event-by-bond or event-by-pair fixed effects and cluster standard errors by event. Coefficients are post-event changes in basis points.

Table 20: Pass-through of public green-bond events to green and conventional bonds

Event group	Outcome	Window	Events	Obs.	Post	S.E.
All public events	Green ASW	6m	3	140	5.54	–
All public events	Green yield	6m	3	140	0.70	–
All public events	Green z-spread	6m	3	140	4.96	–
All public events	ASW residual	6m	3	140	-0.59	–
All public events	Yield residual	6m	3	140	-0.00	–
All public events	Z-spread residual	6m	3	140	-0.58	–
All public events	Matched ASW	6m	13	253	3.76	3.85
All public events	Matched yield	6m	13	253	0.13	0.18
All public events	Matched z-spread	6m	13	253	3.76	3.52
Framework/SPO	Green ASW	6m	2	94	-0.80	–
Framework/SPO	Green yield	6m	2	94	0.25	–
Framework/SPO	Green z-spread	6m	2	94	-0.56	–
Framework/SPO	ASW residual	6m	2	94	0.13	–
Framework/SPO	Yield residual	6m	2	94	0.00	–
Framework/SPO	Z-spread residual	6m	2	94	0.17	–
Framework/SPO	Matched ASW	6m	5	114	-0.67	–
Framework/SPO	Matched yield	6m	5	114	0.25	–
Framework/SPO	Matched z-spread	6m	5	114	-0.39	–
Reporting	Green ASW	6m	1	46	–	–
Reporting	Green yield	6m	1	46	–	–
Reporting	Green z-spread	6m	1	46	–	–
Reporting	ASW residual	6m	1	46	–	–
Reporting	Yield residual	6m	1	46	–	–
Reporting	Z-spread residual	6m	1	46	–	–
Reporting	Matched ASW	6m	8	139	4.39	4.39
Reporting	Matched yield	6m	8	139	0.11	0.21
Reporting	Matched z-spread	6m	8	139	4.35	4.00

Notes: The table stacks monthly windows around audited green-bond framework, second-party-opinion, allocation, and impact-reporting events. Coefficients are event-level post-minus-pre changes, averaged across green-conventional pairs and then across events. Standard errors are reported only when there are at least three usable event-level paired changes. Green and matched outcomes report issuer-bond yield or spread levels; residual rows report the corresponding green-minus-counterfactual greenium object.

avoids presenting artificial precision in small event cells.

B.4 Framework, Credibility, and Allocation Coding

The credibility index records whether the issuer publishes a framework, second-party opinion, external assurance, allocation report, impact report, machine-readable allocation or impact data, taxonomy alignment, adaptation or resilience content, and quantitative impact metrics. Allocation files are parsed into category shares and accounting timing shares when official reports permit that split. Accounting timing is not causal additionality. It is a proxy for whether proceeds are linked to prior expenditure or current/forward-looking spending.

Table 21: Public credibility scores and country greeniums

Issuer	Score	Curve mean	Matched mean	Events
Austria	8/9	-6.0	-0.0	3
Belgium	9/9	14.7	7.8	4
Brazil	7/9	–	–	0
Canada	9/9	–	–	0
Chile	7/9	–	–	0
Denmark	9/9	2.9	3.3	2
France	3/9	–	–	0
Germany	9/9	1.7	2.4	3
Hong Kong	7/9	-7.7	-27.3	5
Ireland	9/9	–	–	0
Italy	1/9	–	–	0
Netherlands	8/9	-0.9	-1.1	9
New Zealand	5/9	–	–	0
Peru	6/9	–	–	0
Spain	7/9	–	–	0
UK	9/9	–	6.9	3
Uruguay	6/9	–	–	0
World Bank	1/9	–	–	0

Notes: Scores add indicators for framework publication, second-party opinion, external assurance, allocation reporting, impact reporting, machine-readable allocation or impact files, taxonomy alignment, adaptation or resilience content, and quantitative impact metrics. Country means are descriptive and should not be read as causal estimates.

Table 21 reports public credibility scores and descriptive country-level greenium means.

Table 22 reports cross-country mechanism regressions using credibility, adaptation share, current/forward allocation, and the accounting-additionality proxy. These regressions include year fixed effects but not country fixed effects because the program variables are mostly issuer-level characteristics. They should be interpreted as descriptive evidence, not causal estimates.

B.5 Climate-Risk and Disaster Diagnostics

Table 23 uses forward-looking FactSet/LSEG climate-risk variables. With a small issuer-country sample, the slopes are descriptive. They do not support a simple monotone relationship between higher climate-risk scores and larger bond-level greeniums.

Table 22: Additionality and credibility mechanisms in bond-level greeniums

Outcome	Program variable	Obs.	Countries	Coef.	S.E.
Yield greenium	Credibility score	708	4	38.90	33.29
Yield greenium	Adaptation share	708	4	77.94	68.61
Yield greenium	Current/forward allocation	708	4	-3.47	3.34
Yield greenium	Accounting additionality proxy	708	4	-3.47	3.34
ASW greenium	Credibility score	711	4	-308.24	667.70
ASW greenium	Adaptation share	711	4	-600.65	1372.23
ASW greenium	Current/forward allocation	711	4	24.56	62.83
ASW greenium	Accounting additionality proxy	711	4	24.56	62.83
Z-spread greenium	Credibility score	711	4	-512.51	404.32
Z-spread greenium	Adaptation share	711	4	-1039.75	830.66
Z-spread greenium	Current/forward allocation	711	4	43.14	37.43
Z-spread greenium	Accounting additionality proxy	711	4	43.14	37.43

Notes: Program variables are mostly cross-sectional at the issuer level, so these regressions include year fixed effects but not country fixed effects. The coefficients therefore summarize whether countries with more observable credibility, adaptation content, and forward/current allocation have different bond-level residual greeniums, conditional on maturity gap, bid-ask-gap difference, rating, and ND-GAIN vulnerability.

Table 23: Country climate-risk heterogeneity in matched greeniums

Outcome	Country variable	Countries	Slope	Corr.
Yield	climate risk score	12	-0.008	-0.546
Yield	transition risk score	12	-0.005	-0.510
Yield	2050 physical risk score	12	-0.006	-0.374
ASW	climate risk score	12	-0.605	-0.341
ASW	transition risk score	12	-0.192	-0.173
ASW	2050 physical risk score	12	-0.748	-0.409
z-spread	climate risk score	12	-0.176	-0.289
z-spread	transition risk score	12	-0.066	-0.173
z-spread	2050 physical risk score	12	-0.168	-0.268

Notes: Country-level descriptive regressions of average matched greenium residuals on latest FactSet/LSEG climate variables. The small country count means this table is descriptive heterogeneity evidence, not an identification design.

Table 24: State-dependent sovereign greeniums and commitment mechanisms

Outcome	Mechanism interaction	Obs.	Countries	Coef.	S.E.
Yield greenium	Vuln. x credibility	879	8	-0.288***	0.071
Yield greenium	Vuln. x adaptation	835	7	-0.262	0.327
Yield greenium	Climate x current/fwd	708	4	-1.242***	0.434
ASW greenium	Vuln. x credibility	882	8	2.126	4.855
ASW greenium	Vuln. x adaptation	838	7	-57.952***	11.288
ASW greenium	Climate x current/fwd	711	4	14.894***	3.628
Z-spread greenium	Vuln. x credibility	882	8	-0.858	1.749
Z-spread greenium	Vuln. x adaptation	838	7	-24.938***	6.839
Z-spread greenium	Climate x current/fwd	711	4	4.379**	1.943

Notes: The sample is the maturity-caliper matched monthly panel. Reported coefficients are the named interaction terms. ND-GAIN vulnerability is higher for more vulnerable countries; credibility and allocation shares come from official green-bond documents. Specifications include year fixed effects, the sovereign rating score, and the green-minus-conventional bid-ask gap; standard errors are clustered by issuer country. These are descriptive mechanism estimates, not causal country fixed-effect estimates.

Table 24 reports interaction regressions from the merged climate-commitment panel. It combines matched bond prices with public credibility scores, allocation proxies, ND-GAIN vulnerability/readiness, FactSet/LSEG climate variables, and realized EM-DAT disaster exposure.

Table 25: Recent climate disasters and matched sovereign greeniums

Outcome	EM-DAT exposure	Obs.	Countries	Coef.	Cluster s.e.
Yield	Climate events, lag 1 year	1140	12	0.020	(0.029)
Yield	Any event, lag 1-3 years	1140	12	-0.021	(0.135)
Yield	Log damage, lag 1-3 years	1140	12	0.001	(0.026)
Yield	Log affected, lag 1-3 years	1140	12	-0.068	(0.023)
Yield	Events, lag 1-3 years + controls	1038	11	0.015	(0.012)
ASW	Climate events, lag 1 year	1134	12	0.351	(0.399)
ASW	Any event, lag 1-3 years	1134	12	-6.592	(2.540)
ASW	Log damage, lag 1-3 years	1134	12	-2.001	(1.578)
ASW	Log affected, lag 1-3 years	1134	12	-1.032	(0.598)
ASW	Events, lag 1-3 years + controls	1043	11	0.147	(0.312)
Z-spread	Climate events, lag 1 year	1134	12	0.018	(0.273)
Z-spread	Any event, lag 1-3 years	1134	12	-8.172	(1.364)
Z-spread	Log damage, lag 1-3 years	1134	12	-1.785	(0.651)
Z-spread	Log affected, lag 1-3 years	1134	12	-0.899	(0.594)
Z-spread	Events, lag 1-3 years + controls	1043	11	-0.043	(0.229)

Notes: The sample is the monthly individual-bond matched panel with maturity gap no larger than two years. Outcomes are defined so positive values indicate green-bond richness. EM-DAT exposure is constructed from hydrological, meteorological, and climatological disasters, aggregated by country-year, and lagged so that bond prices in year t are matched only to disasters observed through year $t-1$. All specifications include country and calendar-year fixed effects. Standard errors are clustered by issuer country. The controls specification adds the green-minus-conventional bid-ask gap, the sovereign rating score, and the FactSet/LSEG climate-risk score.

Table 25 uses lagged realized disasters from EM-DAT. Recent disaster exposure does not overturn the main greenium result once liquidity, ratings, and forward-looking climate-risk controls are included.

Table 26: Climate-disaster shocks and green-program design

Outcome	Shock	Term	Obs.	Countries	Coef.	S.E.
Yield greenium	Events	Climate events	835	7	-0.005	0.030
Yield greenium	Events	Events x credibility	835	7	-0.076***	0.020
Yield greenium	Events	Events x adaptation	835	7	0.019	0.039
Yield greenium	Damage	Log damage	835	7	-0.005	0.012
Yield greenium	Damage	Damage x credibility	835	7	-0.089***	0.023
Yield greenium	Damage	Damage x adaptation	835	7	-0.037***	0.011
ASW greenium	Events	Climate events	838	7	-4.520*	2.614
ASW greenium	Events	Events x credibility	838	7	-4.236	4.531
ASW greenium	Events	Events x adaptation	838	7	-3.394	3.741
ASW greenium	Damage	Log damage	838	7	1.938***	0.422
ASW greenium	Damage	Damage x credibility	838	7	-3.684	2.560
ASW greenium	Damage	Damage x adaptation	838	7	-2.402	1.504
Z-spread greenium	Events	Climate events	838	7	-1.835	1.790
Z-spread greenium	Events	Events x credibility	838	7	-3.267	2.552
Z-spread greenium	Events	Events x adaptation	838	7	-2.820	1.944
Z-spread greenium	Damage	Log damage	838	7	1.862***	0.265
Z-spread greenium	Damage	Damage x credibility	838	7	-3.569***	1.060
Z-spread greenium	Damage	Damage x adaptation	838	7	-2.667***	0.602

Notes: The regressions interact realized EM-DAT country-year disaster exposure with pre-existing program characteristics. Unlike the level tests, these specifications include country and year fixed effects, so identification comes from within-country changes in realized disaster exposure and cross-country differences in green-program design. Standard errors are clustered by issuer country.

Table 26 uses contemporaneous country-year EM-DAT shocks interacted with pre-existing program design. These specifications include country and year fixed effects, so identification comes from within-country changes in realized disaster exposure scaled by cross-country differences in green-program design.

B.6 Data Gaps

Table 27 records unresolved ISIN-level FactSet gaps. The remaining empirical limitations are less about missing local files and more about identification: the sovereign green-bond universe is small, many mechanism variables are issuer-level and persistent, and public framework/reporting events are often anticipated.

C Computation Appendix

The model is solved by iterating jointly on value functions and long-term bond prices. The production solver is the Fortran probability-window solver called from Julia calibration routines. Julia parallelizes over candidate parameter vectors; each Fortran solve uses one thread to avoid nested parallelism.

Table 27: Priority missing ISIN-date yield data

ISIN	Country	Currency	Side	Pair
AT0000A30663	Austria	EUR	green	210
AT0000A356X8	Austria	EUR	green	211
AT0000A398C2	Austria	EUR	green	212
XS3068732612	Austria	Euro	green	74
IDJ000026109	Indonesia	Indonesian Rupiah	green	3
IDJ000028709	Indonesia	Indonesian Rupiah	green	7
IDJ000030903	Indonesia	Indonesian Rupiah	green	4
IDJ000033709	Indonesia	Indonesian Rupiah	green	8
IDJ000035902	Indonesia	Indonesian Rupiah	green	2
NL0015001RG9	Netherlands	EUR	green	216
CH0557779136	Switzerland	Swiss Franc	green	147
IDJ000015003	Indonesia	Indonesian Rupiah	green	1

Notes: These ISINs appear in the project matching inputs but still lack usable YTM and duration observations in the current FactSet workbook. Pulling these fields would expand the matched-pair and curve-residual designs.

C.1 Taste-Shock Smoothing

EV1 taste shocks smooth default and repayment action choices. This improves numerical stability in the long-term debt price recursion. The deterministic model is approached as $\rho_D, \rho_A \to 0$. The paper treats ρ_D and ρ_A as numerical smoothing parameters and reports robustness.

C.2 Price Recursion

The price update uses repayment probabilities and action-probability-weighted continuation prices. The numerical validation checks default-free prices, price bounds, monotonicity of policy rules, no-enforcement limits, and taste-shock robustness.

C.3 Acceleration

The Fortran solver contains probability-window and guarded-window search routines. Brute-force comparisons on smaller grids validate that the accelerated routines preserve the relevant value, price, default probability, and moment objects. The production baseline uses the conservative probability-window solver.

C.4 Run Provenance

For reproducibility, production runs write the command, environment, git status, grid, solver, tolerance, parameter vector, moments, loss components, and logs to a timestamped run directory. Smoke tests and pilot runs use separate output directories.

D Robustness Appendix

The robustness analysis is organized along four dimensions.

D.1 Quantitative Robustness

The quantitative robustness checks are:

- no-enforcement label: $\psi_0 = \psi_1 = 0$;
- taste shocks: alternative ρ_D, ρ_A;
- climate severity and fiscal-loss split;
- green-obligation intensity χ_G;
- current-use timing η_m;
- green issuance capacity cost γ_G;
- adaptation implementation cost γ_I;
- grid and solver validation.

D.2 Empirical Robustness

The empirical robustness checks are:

- maturity calipers;
- winsorization and outlier trimming;
- Europe/Americas split;
- exclusion of benchmark bonds;
- yield versus ASW versus z-spread measures;
- CDS windows;
- MOVE controls;
- leave-one-country-out estimates.

D.3 Climate Process

The climate-process robustness varies EM-DAT damage thresholds, affected-population fallback thresholds, episode-merging windows, high-state duration rules, and the mapping from high-state losses to output versus fiscal costs.

References

Agliardi, Elettra and Rossella Agliardi, "Pricing Climate-Related Risks in the Bond Market," *Journal of Financial Stability*, 2019, *44*, 100687.

— **and** — , "Corporate Green Bonds: Understanding the Greenium in a Two-Factor Structural Model," *Environmental and Resource Economics*, 2021, *80*, 257–278.

Aguiar, Mark and Gita Gopinath, "Defaultable Debt, Interest Rates and the Current Account," *Journal of International Economics*, 2006, *69* (1), 64–83.

— **, Manuel Amador, Hugo Hopenhayn, and Iván Werning**, "Take the Short Route: Equilibrium Default and Debt Maturity," *Econometrica*, 2019, *87* (2), 423–462.

Ando, Sakai, Chenxu Fu, Francisco Roch, and Ursula Wiriadinata, "How Large is the Sovereign Greenium?," *Oxford Bulletin of Economics and Statistics*, 2024. Forthcoming; also IMF Working Paper 2023/080.

Arellano, Cristina, "Default Risk and Income Fluctuations in Emerging Economies," *American Economic Review*, 2008, *98* (3), 690–712.

— **and Ananth Ramanarayanan**, "Default and the Maturity Structure in Sovereign Bonds," *Journal of Political Economy*, 2012, *120* (2), 187–232.

Bachelet, Maria Jua, Leonardo Becchetti, and Stefano Manfredonia, "The Green Bonds Premium Puzzle: The Role of Issuer Characteristics and Third-Party Verification," *Sustainability*, 2019, *11* (4), 1098.

Baker, Malcolm, Daniel Bergstresser, George Serafeim, and Jeffrey Wurgler, "Financing the Response to Climate Change: The Pricing and Ownership of US Green Bonds," *Brookings Papers on Economic Activity*, 2018.

Beirne, John, Nuobu Renzhi, and Ulrich Volz, "Feeling the Heat: Climate Risks and the Cost of Sovereign Borrowing," *International Review of Economics & Finance*, 2021, *76*, 920–936.

Boitan, Iustina Alina and Kamilla Marchewka-Bartkowiak, "Climate Change and the Pricing of Sovereign Debt: Insights from European Markets," *Research in International Business and Finance*, 2022, *62*, 101685.

Cappiello, Lorenzo, Gianluigi Ferrucci, Angela Maddaloni, and Veronica Veggente, "Creditworthy: Do Climate Change Risks Matter for Sovereign Credit Ratings?," Technical Report, ECB Working Paper No. 3042 2025.

Cevik, Serhan and João Tovar Jalles, "This Changes Everything: Climate Shocks and Sovereign Bonds," *IMF Working Paper 2020/079*, 2020.

Chatterjee, Satyajit and Burcu Eyigungor, "Maturity, Indebtedness, and Default Risk," *American Economic Review*, 2012, *102* (6), 2674–2699.

Cheng, Gong, Torsten Ehlers, Frank Packer, and Yanzhe Xiao, "Sovereign Green Bonds: A Catalyst for Sustainable Debt Market Development?," Technical Report, BIS Working Paper 1198 / IMF Working Paper 2024/120 2024.

D'Amico, Stefania, Johannes Klausmann, and N. Aaron Pancost, "The Benchmark Greenium," *Journal of Financial Economics*, 2025. Forthcoming.

Doronzo, Raffaele, Vittorio Siracusa, and Fabrizio Ferriani, "Green Bonds: The Sovereign Issuers' Perspective," *Bank of Italy Markets, Infrastructure, Payment Systems Working Paper*, 2021.

Eaton, Jonathan and Mark Gersovitz, "Debt with Potential Repudiation: Theoretical and Empirical Analysis," *Review of Economic Studies*, 1981, *48* (2), 289–309.

Fama, Eugene F and Kenneth R French, "Disagreement, Tastes, and Asset Prices," *Journal of Financial Economics*, 2007, *83* (3), 667–689.

Fatica, Serena, Roberto Panzica, and Michela Rancan, "The Pricing of Green Bonds: Are Financial Institutions Special?," *Journal of Financial Stability*, 2021, *54*, 100873.

Flammer, Caroline, "Corporate Green Bonds," *Journal of Financial Economics*, 2021, *142* (2), 499–516.

Giglio, Stefano, Bryan Kelly, and Johannes Stroebel, "Climate Finance," *Annual Review of Financial Economics*, 2021, *13*, 15–36.

Grzegorczyk, Marta and Guntram Wolff, "Greeniums in Sovereign Bond Markets," Technical Report, Bruegel Working Paper 2022.

Hachenberg, Britta and Dirk Schiereck, "Are Green Bonds Priced Differently from Conventional Bonds?," *Journal of Asset Management*, 2018, *19*, 371–383.

Halac, Marina and Pierre Yared, "Fiscal Rules and Discretion under Persistent Shocks," *Econometrica*, 2014, *82* (5), 1557–1614.

_ and _ , "Fiscal Rules and Discretion in a World Economy," *American Economic Review*, 2018, *108* (8), 2305–2334.

Hatchondo, Juan Carlos and Leonardo Martinez, "Long-Duration Bonds and Sovereign Defaults," *Journal of International Economics*, 2009, *79* (1), 117–125.

_ , _ , and César Sosa-Padilla, "Debt Dilution and Sovereign Default Risk," *Journal of Political Economy*, 2016, *124* (5), 1383–1422.

_ , _ , and Francisco Roch, "Fiscal Rules and the Sovereign Default Premium," *American Economic Journal: Macroeconomics*, 2022, *14* (4), 244–273.

Hong, Harrison, G Andrew Karolyi, and José A Scheinkman, "Climate Finance," *Review of Financial Studies*, 2020, *33* (3), 1011–1023.

Kapraun, Julia, Carmelo Latino, Christopher Scheins, and Christian Schlag, "(In)-Credibly Green: Which Bonds Trade at a Green Bond Premium?," *Proceedings of Paris December 2019 Finance Meeting*, 2021.

Kling, Gerhard, Ulrich Volz, Victor Murinde, and Sibel Ayas, "Climate Vulnerability and the Cost of Debt," *World Development*, 2021, *145*, 105530.

Klusak, Patrycja, Matthew Agarwala, Matt Burke, Moritz Kraemer, and Kamiar Mohaddes, "Rising Temperatures, Falling Ratings: The Effect of Climate Change on Sovereign Creditworthiness," *Management Science*, 2023, *69* (12), 7468–7491.

Krueger, Philipp, Zacharias Sautner, and Laura T Starks, "The Importance of Climate Risks for Institutional Investors," *Review of Financial Studies*, 2020, *33* (3), 1067–1111.

Mallucci, Enrico, "Natural Disasters, Climate Change, and Sovereign Risk," *Journal of International Economics*, 2022, *139*, 103672.

Panizza, Ugo, Beatrice Weder di Mauro, Sherry Shi, and Mitu Gulati, "The Sovereign Greenium: Big Promise but Small Price Effect," Technical Report, CEPR Discussion Paper No. 20817 2025.

Pástor, Ľuboš, Robert F Stambaugh, and Lucian A Taylor, "Sustainable Investing in Equilibrium," *Journal of Financial Economics*, 2021, *142* (2), 550–571.

Tang, Dragon Yongjun and Yupu Zhang, "Do Shareholders Benefit from Green Bonds?," *Journal of Corporate Finance*, 2020, *61*, 101427.

Zenios, Stavros A, "The Risks from Climate Change to Sovereign Debt," *Climatic Change*, 2022, *172* (3), 1–19.

Zerbib, Olivier David, "The Effect of Pro-Environmental Preferences on Bond Prices: Evidence from Green Bonds," *Journal of Banking & Finance*, 2019, *98*, 39–60.