Quantitative Easing and Sovereign debt Maturity Choice*

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Abstract

The widespread use of unconventional monetary policies has fundamentally altered the environment for sovereign debt management. This paper investigates how sovereign governments should strategically manage their debt maturity structure when central banks are active participants in bond markets. Using Spanish data from 2009-2024, we provide empirical evidence that the Treasury systematically lengthens debt maturity in response to European Central Bank asset purchases. To rationalize this behavior, we develop a quantitative sovereign debt model with default risk where the government issues both short-term and long-term debt, and a central bank actively manages a bond portfolio. The key innovation is a portfolio balance channel where the term premium on long-term debt endogenously responds to the net supply of bonds held by private investors. The calibrated model successfully reproduces the empirical relationship: the long-term debt fraction rises as central bank holdings increase. Counterfactual analysis confirms that eliminating the variable term premium removes this systematic relationship entirely. Comparing a *Strategic Treasury* to a passive government constrained to fixed issuance mix reveals nuanced welfare trade-offs. The activist approach achieves consumption smoothing and benefits from strategic market timing, but exhibits higher default risk and lower sustainable debt levels.

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1 Introduction

The financial crisis of 2008 and the subsequent European sovereign debt crisis fundamentally transformed the landscape of monetary policy and sovereign debt management. Central banks worldwide adopted unconventional monetary policies, most notably Quantitative Easing (QE), becoming major participants in government bond markets through large-scale asset purchase programs. This unprecedented intervention has raised critical questions about how sovereign debt managers should optimally respond to central bank activities in bond markets, yet the strategic interaction between monetary and fiscal authorities in this new environment remains poorly understood.

This paper investigates a fundamental question: How should a sovereign government strategically manage its debt maturity structure when the central bank is an active participant in the bond market? We address this question through both empirical analysis and theoretical modeling, focusing on the portfolio balance channel[1] through which central bank asset purchases affect the term structure of interest rates and, consequently, the relative attractiveness of different debt maturities for sovereign issuers.

This study addresses the question of how sovereign governments should strategically manage their debt maturity structure when central banks pursue unconventional monetary policies. The framework developed here demonstrates that Treasuries are not merely cost minimizers, but can act as *strategic treasuries*, using maturity policy to complement or offset central bank actions. This insight reframes debt management as a crucial and dynamic aspect of fiscal-monetary coordination in advanced economies.

The motivation for this research stems from several key observations. First, the scale of central bank interventions has been substantial. The European Central Bank's Public Sector Purchase Programme (PSPP) and Pandemic Emergency Purchase Programme (PEPP) resulted in holdings of approximately €500 billion in Spanish government bonds by late 2022, representing a significant fraction of the outstanding debt stock. Second, preliminary evidence suggests that sovereign debt managers have not remained passive observers of these interventions. Rather, there appears to be a systematic relationship between central bank asset purchases and changes in the maturity composition of new sovereign debt issuance. Recent observations in the United States suggest a notable deviation from traditional, passive debt issuance strategies. This shift, has sparked debate about the government's role in counteracting the central bank's monetary tightening. While the central bank raises policy rates to curb inflation, the Treasury's preference for short-term bills may ease financial conditions, mimicking the effects of quantitative easing (QE). This paper explores this strategic fiscal policy tool, examining how the Treasury manipulates debt maturity to influence term premia, complementing or offsetting central bank actions.

We propose and test an "Activist Treasury" framework, in which sovereign debt managers strategically adjust their issuance patterns in response to central bank interventions. The central hypothesis is that during QE periods, when large-scale asset purchases reduce long-term yields through portfolio balance effects, treasuries favor long-term issuance to lock in favorable rates and mitigate future refinancing risks. Conversely, during Quantitative Tightening (QT), when central banks reduce their

[1] EXPLAIN THAT THIS IS NOT THE ONLY MECHANISM AND ADD SOURCES

holdings through run-offs or sales, the increased net supply of bonds to private investors elevates yields, potentially incentivizing a shift toward short-term debt to avoid higher long-term borrowing costs.

The challenge lies in the fact that the traditional separation of roles between an independent central bank focused on price stability and a treasury focused on financing the government has become blurred. Recent issuance patterns by fiscal authorities, such as the U.S. Treasury, suggest a strategic departure from passive debt management, fueling a debate about the potential for an "Activist Treasury" – fiscal authority that strategically alters its debt issuance mix to influence term premia and broader financial conditions. This raises a crucial, unanswered question: How should a sovereign government optimally manage the maturity structure of its debt when the central bank is actively intervening in the bond market? Answering this question is challenging because it requires a framework that can simultaneously capture the sovereign's default risk, the trade-offs between short- and long-term debt, and the direct impact of central bank actions on the term structure of interest rates.

This paper addresses this question by developing a quantitative framework to analyze the optimal maturity choice of a sovereign that faces default risk in the presence of an active central bank. Our central contribution is to model and quantify the strategic behavior of an "Activist Treasury," demonstrating that the flexibility to adjust debt maturity in response to monetary policy is a valuable tool for macroeconomic stabilization.

To ground our analysis, we first establish an empirical regularity that motivates our theoretical investigation. Our empirical investigation focuses on Spain, which provides an ideal laboratory for this analysis. As a high-debt Eurozone economy with public debt averaging around 100% of GDP over our sample period, Spain is particularly sensitive to European Central Bank (ECB) policies due to its reliance on external financing and the absence of national monetary autonomy. Using monthly data from 2009 to 2024, we conduct a time-series analysis of Spain's debt issuance patterns and their relationship to ECB asset holdings. Our empirical strategy employs a fractional logit model to analyze changes in the share of short-term debt in new issuance, appropriately accounting for the bounded nature of this variable.

The empirical results provide strong support for the Activist Treasury hypothesis. We find that increases in ECB net purchases are associated with a systematic lengthening of debt maturity by the Spanish Treasury. Specifically, a one percentage point of GDP increase in ECB net purchases is associated with a decrease of nearly one percentage point in the probability of an increase in the short-term issuance share. This relationship is robust to various model specifications and control variables, including the overall monetary policy stance and yield curve dynamics.

To rationalize these empirical findings and explore their broader implications, we develop a quantitative sovereign debt model that extends the canonical framework of Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012). The key innovation is the integration of a central bank that actively manages a portfolio of government bonds, influencing the term premium on long-term debt through a portfolio balance channel. In our model, while international investors are largely risk-neutral, a specialized segment has a preferred habitat for long-term bonds and must be compensated with a premium to deviate from their preferred holdings. Central bank purchases reduce the net supply of long-term bonds to private investors, thereby lowering this term premium and making long-term debt relatively more attractive to the sovereign issuer.

The model, calibrated to Spanish data, including European Central Bank (ECB) balance sheet dynamics and Treasury issuance patterns, enabling the Treasury to strategically influence long-term yields by adjusting the net supply of different maturities available to private investors, lowering term premia and easing financial conditions. The model generates several important predictions. First, the model successfully rationalizes the empirical evidence. The core economic mechanism is the term premium channel: when the central bank implements QE, it absorbs a significant portion of long-term bonds from the market. This reduction in the net supply to private investors compresses the term premium, making long-term debt a relatively cheaper source of financing. The *Activist Treasury* in our model strategically exploits this opportunity to lock in favorable long-term borrowing costs, tilting its issuance towards longer maturities. Our simulations show that the average long-term debt fraction in new issuance rises from 31.8% to 40.6% as central bank holdings increase from their lowest to highest quintile, generating a negative correlation between the short-term issuance share and central bank holdings that is consistent with our empirical estimates.

To isolate the importance of the portfolio balance channel, we conduct a counterfactual analysis comparing the benchmark model with an alternative specification where the term premium is exogenously fixed. This comparison reveals that when the term premium does not respond to central bank actions, the systematic relationship between central bank holdings and debt maturity vanishes entirely. This finding underscores that it is specifically the sensitivity of the term premium to central bank operations that provides the key incentive for activist debt management.

Second, we demonstrate the significant economic value of this strategic flexibility through a counterfactual analysis. We compare the outcomes of our benchmark "Activist Treasury" to those of a "passive" government constrained to a fixed issuance mix. Surprisingly, we find that the welfare implications are nuanced, with each approach offering distinct advantages and disadvantages. The activist Treasury achieves some consumption smoothing, with a correlation between consumption and income of 0.924 compared to perfect correlation (1.0) under a passive policy, and benefits from lower average term premia (258 basis points versus 499 basis points) through strategic market timing. However, this flexibility comes at significant costs: the activist approach exhibits procyclical borrowing patterns, higher default risk (0.258% versus 0%), and lower sustainable debt levels.

Paradoxically, the passive Treasury—despite facing higher term-premium costs and lacking maturity adjustment flexibility—sustains higher debt levels (66.8% versus 63.0% of GDP) at zero default risk. This counterintuitive result emerges from two key mechanisms. First, the fixed and low fraction of short-term debt issuance reduces rollover risk, eliminating default risk and thereby raising asset prices. Second, the constrained debt mix leads to lower consumption volatility relative to output, and this aggregate stability reduces the sovereign's willingness to default, further supporting higher asset prices through lower default risk. These findings reveal a fundamental trade-off in debt management: strategic flexibility offers the ability to exploit favorable market conditions and achieve some consumption smoothing, but at the cost of higher volatility and default risk that ultimately constrains the sovereign's borrowing capacity.

This research makes several important contributions to the literature. First, it provides novel empirical evidence on the strategic interaction between monetary and fiscal authorities in advanced economies, complementing the extensive literature on emerging market sovereign debt that has traditionally focused on default risk as the primary driver of maturity choices. Second, it introduces a theoretical framework that incorporates central bank balance sheet policies into sovereign debt models, filling an important gap in our understanding of fiscal-monetary interactions in the era of unconventional monetary policy.

The findings of this paper carry important policy implications. They suggest that sovereign debt management is not merely a technical exercise in financing the government deficit but can be a powerful tool for macroeconomic policy. The strategic actions of an "Activist Treasury" can either amplify or dampen the effects of unconventional monetary policy, highlighting the need for close coordination between fiscal and monetary authorities. During QE, a Treasury that lengthens its debt maturity can enhance the central bank's stimulus by further reducing long-term yields. Conversely, during QT, the Treasury's issuance decisions could either lean against or exacerbate monetary tightening, with significant consequences for financial conditions and economic stability.

Furthermore, our research speaks to the broader debate about the coordination of monetary and fiscal policy. While much of the existing literature focuses on the fiscal theory of the price level and the potential for fiscal dominance, our work highlights a different dimension of fiscal-monetary interaction: the strategic complementarity between central bank asset purchases and sovereign debt management. This complementarity suggests that effective policy coordination may require not just agreement on macroeconomic objectives but also active coordination of operational decisions regarding asset purchases and debt issuance.

Literature review This paper is positioned at the intersection of three distinct but complementary fields of macroeconomic research: the quantitative theory of sovereign default, the literature on the portfolio balance channel of unconventional monetary policy, and the policy-oriented debate on strategic debt management. Our central contribution is to bridge the gaps between these literatures by developing the first quantitative framework that integrates a strategic, defaultable sovereign with an active central bank that influences the term premium.

The modern quantitative literature on sovereign debt, originating with the seminal work of Eaton and Gersovitz (1981), models default as a strategic choice by a government weighing the benefits of repudiation against the costs of market exclusion and output losses. Foundational quantitative models by Arellano (2008) and Aguiar and Gopinath (2006) successfully replicated key business cycle facts for emerging markets by focusing on credit risk as the sole determinant of bond prices. A crucial extension, pioneered by Hatchondo and Martinez (2009a) and Chatterjee and Eyigungor (2012a), incorporated long-term debt, introducing a richer trade-off between rollover risk (from short-term debt) and debt dilution (from long-term debt). Arellano and Ramanarayanan (2012) emphasize the role of rollover risk, showing that governments prefer longer maturities during periods of high volatility to avoid frequent refinancing in potentially adverse market conditions. These models are structurally unable to analyze the primary effects of Quantitative Easing (QE), as there is no mechanism for central bank purchases to affect the term premium. By incorporating an active central bank as a major bond market participant, this paper moves beyond the traditional focus on default risk to examine how portfolio balance effects shape optimal debt maturity in advanced economies with low default risk. Our model demonstrates that strategic issuance behavior – varying the maturity of debt in response to monetary policy – can improve financing conditions but may also increase default risk and limit sustainable debt levels. These mechanisms are absent from existing sovereign default models.

In parallel, the literature on QE and the term premium has emphasized the importance of central bank asset purchases in flattening the yield curve through the portfolio balance channel. Empirical evidence strongly supports the portfolio balance channel. D'Amico and King (2013) show that Federal Reserve purchases during QE1 significantly reduced yields on targeted securities, with effects concentrated in purchased bonds rather than spreading uniformly across the yield curve. Their findings demonstrate that portfolio balance effects operate through local supply effects rather than broad signaling channels. Krishnamurthy and Vissing-Jorgensen (2011) extend this analysis, showing QE operates through portfolio balance effects, duration risk removal, and safety premium channels. Swanson (2021) shows that the term premium channel is the dominant mechanism in QE transmission for the Euro area, accounting for most macroeconomic effects. Theoretical support for these findings is provided by preferred-habitat models, most notably Vayanos and Vila (2021), in which segmented bond markets and investor preferences for specific maturities imply that changes in bond supply can directly affect long-term yields. Central bank purchases of long-term bonds reduce the net supply of duration risk available to the market, thereby lowering the excess return required by investors to hold these assets. However, much of this literature abstracts from strategic fiscal behavior. The supply of government bonds is typically assumed exogenous, or QE is analyzed independently of sovereign risk and debt management. The role of the fiscal authority as an active agent responding to changes in yields induced by QE remains underexplored.

A third relevant strand is the literature on fiscal-monetary interactions. The breakdown of strict division between monetary and fiscal policy, especially after the global financial crisis, has drawn attention to the macroeconomic consequences of this policy mix. Bianchi and Melosi (2019) demonstrate how the effectiveness of monetary policy depends on the fiscal regime, while Corsetti et al. (2013) show that central bank interventions can alter the constraints facing fiscal authorities and overall macroeconomic stability. Greenwood et al. (2014) propose that debt management can itself serve as an active macroeconomic instrument, influencing term premia and financial conditions. We provide the first empirical evidence and theoretical framework for strategic debt management responses to central bank asset purchases, demonstrating how activist treasuries can complement or offset monetary policy through maturity choice. Empirically, the paper validates the model's mechanism using data from Spain during the ECB's QE programs between 2009 and 2024. It shows that episodes of large-scale asset purchases are associated with systematic shifts in debt issuance toward longer maturities.

Taken together, this paper offers a novel contribution by linking QE and the term premium literature with sovereign default theory and fiscal behavior. It advances our understanding of how unconventional monetary policy transmits to fiscally constrained economies and provides a new tool to evaluate the macro-financial consequences of activist debt management in the age of QE.

The remainder of the paper is organized as follows. Section (2) presents our empirical analysis of Spanish debt issuance patterns and their relationship to ECB policies. Section (3) develops the theoretical model, while Section (4) details the calibration strategy. Section (5) presents the main theoretical results on optimal debt maturity choice, and Section (6) provides the counterfactual analysis isolating the term premium channel. Section (7) examines the welfare implications of activist versus passive debt management. Section (8) concludes with a discussion of policy implications and directions for future research.

2 Empirical Analysis of Activist Treasury Issuance

2.1 Empirical Strategy and Hypothesis

The widespread implementation of unconventional monetary policies has fundamentally altered the environment for sovereign debt management. Large-scale asset purchase programs, colloquially known as Quantitative Easing (QE), have made central banks major participants in government bond markets, raising important questions about the strategic response of fiscal authorities. This empirical section investigates a key aspect of this new dynamic: how a sovereign treasury adjusts the maturity structure of its debt issuance in response to large-scale purchases of its bonds by the central bank.

We test the hypothesis that a sovereign's debt management is not a passive exercise, but rather an active strategy that responds to and interacts with the central bank's interventions. Specifically, we propose and test an "Activist Treasury" framework, in which the treasury strategically alters its issuance to complement the effects of monetary policy. The central, testable proposition is that during QE periods, characterized by large-scale asset purchases that reduce long-term yields through portfolio balance effects, treasuries might favor long-term issuance to lock in favorable rates and mitigate future refinancing risks. Conversely, QT, involving the reduction of central bank holdings via run-offs or sales, increases net bond supply to private investors, potentially elevating yields and incentivizing a shift toward short-term debt to avoid higher long-term borrowing costs.

This section is dedicated to testing this primary hypothesis empirically. We conduct a time-series analysis of Spain's debt issuance to investigate whether the Treasury's behavior has been consistent with this "Activist" framework. A finding in support of this hypothesis would establish an important empirical regularity and motivate the development of the quantitative model presented in the subsequent sections, which is designed to rationalize such strategic behavior.

Spain provides a compelling context for this analysis, as a high-debt Eurozone economy (with public debt averaging around 100% of GDP over the sample period) that is particularly sensitive to ECB policies due to its reliance on external financing and the absence of national monetary autonomy. The ECB's Public Sector Purchase Programme (PSPP), launched in March 2015 and extended through December 2018 with subsequent reinvestments, along with the Pandemic Emergency Purchase Programme (PEPP) from March 2020 to March 2022, resulted in substantial holdings of Spanish bonds, peaking at approximately €500 billion by late 2022. QT commenced in March 2023 with partial non-reinvestment of maturing securities under the Asset Purchase Programme (APP), escalating to full QT by July 2024 without reinvestments, and further adjustments under PEPP from mid-2024 onward. By focusing on monthly data from January 2015 to December 2023, we capture the full QE expansion and the onset of QT, allowing us to isolate the flow effects of ECB bond holdings while controlling for the broader monetary stance, yield curve dynamics, and fiscal conditions.

2.2 Data and Variable Construction

To formally test our hypothesis, we constructed a monthly dataset for the Spanish economy covering the period from January 2009 to December 2024. This sample of monthly observations is specifically chosen to encompass the initiation and expansion of the ECB's main QE programs (the PSPP and PEPP) and the subsequent transition to Quantitative Tightening (QT), thereby providing the necessary policy variation for our analysis.

The dependent variable is designed to capture the Treasury's active, marginal decisions about debt maturity. It is the share of short-term debt in new issuance, denoted ST_Share_t. This is calculated as the gross monthly issuance of Treasury bills (*Letras del Tesoro*, with maturities of one year or less) divided by the total gross monthly issuance of all government securities. Focusing on the composition of *new* issuance, rather than the total outstanding stock of debt, allows us to directly observe the Treasury's contemporaneous response to market conditions. To analyze the dynamics of this strategic choice and to ensure our time-series variables are stationary, our primary outcome variable is the first difference of a three-month moving average of this share, $\Delta ST\_Share_t^{MA}$. Using a moving average smooths idiosyncratic noise from individual auctions, while differencing focuses the analysis on the change in issuance strategy from one month to the next. Data on gross issuance for all security types are compiled from the official monthly statistical publications of the Tesoro Público de España.

The key explanatory variable, capturing the stance of unconventional monetary policy, is the monthly change in ECB holdings of Spanish government bonds, ΔECB_Holdings_Spain, calculated as the difference in outstanding amounts under PSPP and PEPP. This flow variable proxies the net impact of QE purchases (positive during 2009-2024) and QT run-offs (negative from March 2023). Data are obtained from the ECB's monthly consolidated financial statements and asset purchase reports, available on the ECB website, which detail total holdings. Country-specific holdings for Spain are estimated by multiplying the total ECB asset purchases under PSPP and PEPP by Spain's average Eurosystem capital key over the period 2009-2024, which is approximately 12%. The Eurosystem capital key represents the share of each euro area national central bank in the ECB's capital, adjusted periodically based on population and GDP, and is used to allocate purchases proportionally among euro area countries. To account for economic scale, we normalize by Spain's nominal GDP (from Eurostat), yielding ΔNH_pp in percentage points (multiplied by 100 for interpretability). A 1-unit change corresponds to approximately €13 billion in monthly flows relative to Spain's average annual GDP of €1.3 trillion during the period. It is crucial to use this *flow* measure rather than the total *stock* of ECB holdings. The stock is a slow-moving, highly persistent variable that reflects the accumulation of all past actions, making it difficult to identify the impact of policy changes in a specific month. The flow, in contrast, precisely captures the active intervention—be it a net purchase under QE or a net reduction under QT—in the period it occurs, directly aligning with the theoretical mechanism of altering the net supply of bonds available to the private sector. We normalize this flow by Spain's quarterly nominal GDP from Eurostat to ensure the measure is appropriately scaled to the size of the economy.

Control variables are included to isolate the effect of ECB holdings changes from confounding factors that could influence debt maturity decisions. The ECB's Deposit Facility Rate (DFR), sourced from ECB databases, serves as a proxy for the overall monetary policy stance. As the rate at which banks can deposit excess reserves with the ECB, the DFR influences short-term interest rates and borrowing costs across the economy. A higher DFR signals tighter monetary conditions, which may discourage short-term issuance by making rollover financing more expensive or by signaling reduced liquidity support, potentially leading treasuries to favor longer maturities to lock in rates. We expect a negative association with shifts toward short-term debt, consistent with literature on policy rate impacts on funding structures (e.g., Demiralp et al., 2021).

The yield curve slope is measured as the spread between 10-year and 2-year Spanish government bond yields, obtained from Banco de España data. This variable captures the term structure of interest rates, reflecting expectations about future economic conditions, inflation, and monetary policy. A steeper slope (higher long-term yields relative to short-term) increases the relative cost of long-term debt, incentivizing a shift toward short-term issuance to minimize immediate interest expenses. Conversely, a flatter or inverted curve might encourage long-term issuance. This control is crucial for disentangling QE/QT effects from broader market dynamics, with an expected positive sign on the slope for short-term shifts, aligning with market-timing theories (Baker et al., 2003).

2.3 Econometric Specification

The choice of econometric model is dictated by the nature of our dependent variable. Since $\Delta ST\_Share_t^{MA}$ is a change in a proportion, it is a continuous variable, but it is naturally bounded. A standard Ordinary Least Squares (OLS) regression is ill-suited for this type of data for two main reasons. First, a linear model can produce predicted values that fall outside the logical bounds of the variable, producing predictions that are inconsistent with the variable's definition. Second, the error variance of a fractional variable is typically not constant (i.e., it is heteroskedastic), violating a key assumption of OLS and leading to inefficient estimates and incorrect standard errors.

To address these issues, we employ a fractional logit model, as developed by Papke and Wooldridge (1996). This is the appropriate tool for the task. It is a quasi-maximum likelihood technique that models the conditional expectation using a logistic link function, which mathematically constrains the predicted values to lie within the unit interval. Furthermore, the method is inherently robust to the kind of heteroskedasticity found in fractional data. Our baseline specification is:

$$logit(E[y_t|\mathbf{X}_t]) = \beta_0 + \beta_1 \Delta ECB\_Holdings_t^{GDP} + \beta_2 DFR_t + \beta_3 Yield\_Curve_t \tag{1}$$

where y_t is the scaled $\Delta ST\_Share_t^{MA}$. To ensure our statistical inference is robust to any remaining serial correlation and heteroskedasticity, all reported standard errors are Newey-West corrected.

2.4 Methodological Choices

The empirical strategy incorporates several deliberate choices to enhance identification, robustness, and alignment with theoretical predictions. First, employing flows (ΔNH_pp) rather than stock levels of ECB holdings isolates dynamic policy effects, such as monthly net purchases under QE or run-offs during QT, which directly alter bond supply and yields via portfolio rebalancing (D'Amico and King, 2013). Flows exhibit lower autocorrelation than stocks, mitigating persistence biases, and correspond to ECB's tapered schedules (e.g., €15 billion monthly APP run-offs from March 2023, escalating to full QT).

Second, differencing the moving average of ST_Total focuses on short-run adjustments, accounting for non-stationarity in issuance series (confirmed by augmented Dickey-Fuller tests) and emphasizing policy-induced changes over static levels. The three-month window smooths intra-month auction irregularities, while rescaling preserves the binomial structure without truncation biases.

Third, the binomial GLM with logit link is preferred over linear models (e.g., OLS) for its handling of bounded outcomes and non-constant variance, yielding interpretable marginal effects as changes in predicted probabilities. We evaluate these at the sample mean (μ=0.5) for economic relevance.

Robustness is ensured through extensions: lagged regressors (up to two months) to capture delayed Treasury responses; interactions with a QT dummy (1 from March 2023) for regime heterogeneity; and alternative specifications like fractional probit or undifferenced OLS, which yield consistent signs. Endogeneity concerns are limited by the ECB's rule-based, pro-rata allocations across countries, though we acknowledge potential feedback from Spanish fiscal conditions via unobserved channels.

2.5 Results and Interpretation

The estimation results of the fractional logit model are presented in Table 1. The table displays the estimated coefficients, their robust p-values, and the average marginal effects, which are essential for a clear economic interpretation.

Table 1: Fractional Logit Model Results for the Change in Short-Term Debt Share

Variable	Coefficient	NW p-value	Marginal Effect
$\Delta ECB\_Holdings^{GDP}$	−0.0387	0.0248	−0.0097
Deposit Facility Rate (DFR)	−0.0444	0.0000	−0.0111
Yield Curve Slope	0.0589	0.0005	0.0147
Intercept	−0.9402	0.0000	0.2808

Notes: The dependent variable is the scaled first difference of the 3-month moving average of the monthly share of short-term debt in total issuance. Sample: 2009:M7-2024:M12 (N=186). Newey-West standard errors (4 lags) are used. Marginal effects are calculated at the sample mean.

The results provide strong empirical support for our central hypothesis. The coefficient on our key variable of interest, $\Delta ECB\_Holdings^{GDP}$, is -0.0387 and is statistically significant at the 5% level. The negative sign is the central finding. It indicates that a positive flow of ECB purchases is associated with a negative change in the short-term debt share. This signifies that when the ECB increases its net bond purchases, the Spanish Treasury responds by issuing less short-term debt and, by extension, more long-term debt. This is precisely the behavior predicted by the Activist Treasury framework. The marginal effect provides a sense of the economic magnitude: a one percentage point of GDP increase in ECB net purchases—a realistic monthly flow during active QE—is associated with a decrease of nearly one percentage point in the probability of an increase in the short-term issuance share.

The control variables also behave in an economically intuitive manner, which lends credibility to the model specification. The yield curve slope has a positive and highly significant coefficient, indicating that as the yield curve steepens, making long-term borrowing relatively more expensive, the Treasury increases its share of short-term debt. This finding is consistent with a logical cost-minimizing response. The Deposit Facility Rate has a negative and significant coefficient. This suggests that as conventional monetary policy tightens, the Treasury also tends to shift away from short-term debt, perhaps to lock in funding costs for longer periods in anticipation of further rate hikes.

2.6 Discussion and Conclusion

Our empirical investigation demonstrates that the Spanish Treasury's debt management strategy is not passive but responds dynamically to the monetary policy environment. The key finding—that ECB asset purchases are met with a corresponding lengthening of issuance maturity by the Treasury—provides strong evidence in favor of the "Activist Treasury" hypothesis. This behavior is consistent with the portfolio balance channel, where the Treasury's actions reinforce the ECB's goal of lowering long-term yields.

The robustness of this result was confirmed through several additional tests, including the use of lagged variables and interacting our key variable with a dummy for the QT period, which confirmed an asymmetric response. These findings lend significant empirical weight to the theoretical model that will be developed in the subsequent sections of this paper. They show that the interaction between the central bank's balance sheet and sovereign debt management is not merely a theoretical curiosity but a tangible and important feature of modern macroeconomic policy, with clear implications for the transmission of unconventional monetary policy.

3 A Model of Sovereign Debt with Central Bank Intervention

This section develops a quantitative model of a small open economy where a sovereign government faces default risk. The framework builds upon the canonical sovereign default literature, particularly the models with long-term debt as in Hatchondo and Martinez (2009b) and Chatterjee and Eyigungor (2012b). The key innovation is the integration of a central bank that actively manages a portfolio of government bonds, influencing the term premium on long-term debt through a portfolio balance channel. This allows us to analyze how an "Activist Treasury" strategically manages its debt maturity structure in response to unconventional monetary policies like Quantitative Easing (QE) and Tightening (QT).

3.1 The Economic Environment

Time is discrete and indexed by $t \in \{0, 1, 2, \dots\}$. The economy is populated by a government, a central bank and a continuum of international investors. The Government is benevolent and makes decisions to maximize household's utility.

Government and Preferences

The government maximizes the expected lifetime utility of a representative household. The objective function is given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{2}$$

where E_0 is the expectation operator conditional on information at time $t = 0$, $\beta \in (0, 1)$ is the subjective discount factor, and c_t is private consumption of a single tradable good. The period utility function, $u(c_t)$, is strictly increasing and strictly concave, satisfies the Inada conditions and exhibits constant relative risk aversion (CRRA):

$$u(c_t) = \frac{c_t^{1-\gamma} - 1}{1-\gamma} \tag{3}$$

where $\gamma > 0$ is the coefficient of relative risk aversion. The economy receives a stochastic endowment of the tradable good, y_t, which follows a first-order autoregressive (AR(1)) process in logarithms:

$$\log(y_t) = (1-\rho_y)\mu_y + \rho_y \log(y_{t-1}) + \epsilon_{y,t} \tag{4}$$

where $|\rho_y| < 1$ is the persistence parameter, μ_y is the mean of the process, and $\epsilon_{y,t} \sim \mathcal{N}(0, \sigma_y^2)$ is an i.i.d. shock.

Financial Instruments and Markets

The government can issue two types of non-state-contingent, defaultable bonds to international investors. The first instrument is a short-term bill, a one-period discount bond denoted by $b_{s,t+1}$, which is issued at time t and promises to pay one unit of the consumption good at time $t+1$. The second instrument is a long-term bond, modeled as a consol-like security with geometrically decaying coupons. Let $b_{l,t}$ be the stock of outstanding long-term bonds at the beginning of period t. These bonds require a coupon payment of $\lambda b_{l,t}$ at time t. The stock of long-term bonds evolves according to the law of motion:

$$b_{l,t+1} = (1-\lambda) b_{l,t} + i_{l,t} \tag{5}$$

where $i_{l,t}$ represents new issuances of long-term bonds in period t, and the parameter $\lambda \in (0,1)$ governs the decay rate of coupons, which is inversely related to the bond's duration.

The government's budget constraint in a period when it repays its debt is:

$$c_t + b_{s,t} + \delta b_{l,t} = y_t + q_{s,t} b_{s,t+1} + q_{l,t} i_{l,t} \tag{6}$$

where $b_{s,t}$ and $\lambda b_{l,t}$ are the maturing obligations on short- and long-term debt, respectively. The government finances consumption and debt service with its endowment and the proceeds from new short-term issuance, $q_{s,t} b_{s,t+1}$, and new long-term issuance, $q_{l,t} i_{l,t}$. By substituting the law of motion for long-term bonds from equation (5), the budget constraint can be expressed in terms of the end-of-period debt stocks:

$$c_t + b_{s,t} + \kappa b_{l,t} = y_t + q_{s,t} b_{s,t+1} + q_{l,t}[b_{l,t+1} - (1-\lambda) b_{l,t}] \tag{7}$$

Sovereign Default

External debt service is not enforceable, as the Government can optimally choose to default on external bonds issued in the previous period. If the Government decides to repudiate external debt, the economy faces two types of costs. First, the Government loses the ability to raise external debt for a stochastic number of periods. The government can gain access to external credit markets with probability ξ. Second, during the period of financial autarky the country suffers an output loss. During exclusion periods income is given by $y_t - \phi(y_t)$, where $\phi(y_t)$ is the income loss function. The country re-enters with zero debt outstanding after default.

The Central Bank and the Portfolio Balance Channel

The central bank is an active participant in the government bond market. It holds a stock of the government's long-term bonds, $B_{cb,t}$, which it adjusts over time to implement its monetary policy objectives. We model the central bank's holdings as an exogenous stochastic process that follows an AR(1) in logarithms, capturing the persistent nature of QE and QT policies:

$$\log(B_{cb,t+1}) = (1 - \rho_{cb})\mu_{cb} + \rho_{cb} \log(B_{cb,t}) + \epsilon_{cb,t+1} \tag{8}$$

where $\rho_{cb} \in [0, 1)$ and $\epsilon_{cb,t+1} \sim \mathcal{N}(0, \sigma_{cb}^2)$.

A key feature of our model is a *portfolio balance channel*, inspired by Vayanos and Vila (2012), through which central bank interventions affect the term premium on long-term debt. We assume that while international investors are largely risk-neutral, a specialized segment of these investors has a preferred habitat for long-term bonds. To be induced to hold more of these bonds than their preferred amount, they demand a premium. This term premium, ζ_t, is an increasing function of the net supply of long-term bonds held by the private sector. The central bank's purchases reduce this net supply, thereby lowering the term premium. We specify this relationship with a simple, linear functional form:

$$\zeta_t = \alpha \cdot \max(0, b_{l,t+1} - B_{cb,t+1}) \tag{9}$$

where the parameter $\alpha > 0$ determines the sensitivity of the term premium to the net supply of long-term bonds.

3.2 Recursive formulation

Let the state of the economy be summarized by the vector $S = (b_s, b_l, y, B_{cb})$, representing the outstanding stocks of short- and long-term debt and the current realizations of the endowment and central bank holdings shocks. The government's problem can be formulated recursively.

Value Functions

The value function for a government entering a period with state S, denoted $V(S)$, is the maximum of the value of repaying, $V^R(S)$, and the value of defaulting, $V^D(S)$:

$$V(S) = \max\{V^R(S), V^D(S)\} \tag{10}$$

If the government chooses to repay its debt, its value is given by the solution to the following problem:

$$V^R(S) = \max_{c, b_s', b_l'} \{u(c) + \beta E[V(S')|S]\} \tag{11}$$

subject to the budget constraint from equation (7), where primes denote next-period variables.

$$c + b_s + \kappa b_l = y + q_s(\mathbf{z}, b_s', b_l')b_s + q_l(\mathbf{z}, b_s', b_l') \left[b_l' - (1 - \lambda)b_l\right] \tag{12}$$

If the government chooses to default, it enters a state of financial autarky. Its value is given by:

$$V^D(y, B_{cb}) = u(y - \phi(y)) + \beta E\left[(1-\xi)V^D(y', B'_{cb}) + \xi V(0, 0, y', B'_{cb})|y, B_{cb}\right] \tag{13}$$

Note that the value of default does not depend on the level of debt, as it is entirely repudiated.

Equilibrium Bond Prices

International investors are risk-neutral and price bonds according to their expected discounted payoffs, conditional on the government's future actions. The risk-free discount factor is $1/(1+r)$. The price of a one-period short-term bill, q_s, is the discounted expected probability of repayment:

$$q_s(S, b'_s, b'_l) = \frac{1}{1+r} E\left[1 - d(S')|S\right] \tag{14}$$

where $d(S') \in \{0, 1\}$ is the default indicator function, which equals 1 if the government defaults in state S' and 0 otherwise. The price of a long-term bond, q_l, incorporates the term premium ζ_t into the discount factor. The price is the discounted value of its expected future payoff, which consists of the next period's coupon payment, δ, plus the resale value of the remaining principal, $(1-\delta)q_l(S')$:

$$q_l(S, b'_s, b'_l) = \frac{1}{1+r+\zeta_t} E\left[(1 - d(S'))\left(\delta + (1-\delta)q_l(S', b''_s, b''_l)\right)|S\right] \tag{15}$$

where the term premium $\zeta_t = \alpha \cdot \max(0, b'_l - B'_{cb})$ depends on the chosen end-of-period debt level b'_l and the next period's central bank holdings B'_{cb}.

3.3 Equilibrium definition

A *stationary recursive Markov perfect equilibrium* (SMPE) is defined by a set of functions and rules. These include the value functions $V(S)$, $V^R(S)$, and $V^D(S)$; the policy functions for debt issuance $\hat{b}_s(S)$ and $\hat{b}_l(S)$; the default rule $\hat{d}(S)$; and the bond price functions $q_s(S, b'_s, b'_l)$ and $q_l(S, b'_s, b'_l)$. For an equilibrium to hold, two conditions must be met.

1. Given the price functions,q_s and q_l, the policy functions and the default rule must solve the government's optimization problem as defined by the Bellman equations, (10),(11) and (13).
2. Given the government's policy functions and default rule, the bond price functions must satisfy the investors' break-even conditions, (14) and (15), ensuring rational expectations.

4 Calibration

Externally Determined Parameters. Parameters are categorized into two groups: those derived directly from data and those selected to align model simulations with key data moments. The baseline parameterization is shown in Table 2. The risk-free interest rate is fixed at 0.04, consistent with existing literature. Parameters for the sovereign borrower are based on Spanish data, with each model period representing one quarter of a year.

The utility function employs a constant relative risk aversion coefficient, γ, defined as:$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ The representative agent is assigned a risk aversion coefficient, γ, of 2, value commonly used in studies of business cycles in small open economies. The endowment process, given by Equation **??**, is estimated using Spain's quarterly real GDP data from Q1 1960 to the relevant end period, processed with an HP-filter.

EXPLAIN BCB CALIBRATION the b_{cb} process calibrated to exhibit empirically relevant persistence and to explore a range of values consistent with observed central bank policy stances (details in Appendix X.X).

The probability of the government re-entering foreign credit markets after default, ξ, is set to ensure an average exclusion period of 3 years, following estimates by Richmond and Dias (2009). The sovereign debt recovery rate, α, is calibrated to 0.63, based on Cruces and Trebesch (2013) findings for 1960–2011. The income loss function is modelled quadratically, as in Chatterjee and Eyigungor (2012a), with $\phi(y) = \max\{d_0 y + d_1 y^2, 0\}$, improving the fit to spread data and model equilibrium.

The minimum price for long-term debt, $\underline{q}$, is set at 70% of the average risk-free long-term bond price, limiting the government's debt issuance to a maximum yield to maturity of 9.7% annually. This threshold exceeds the highest yield at which Spain has issued debt since 2008.

Two parameters related to the income cost of default are calibrated to match: (i) the average government debt-to-GDP ratio and (ii) the average long-term interest rate spread, using Spanish data from 2008–2013. This period is selected because interest rate spreads were near zero from 1999–2007, and pre-1999, Spain issued bonds in local currency.

Parameters Calibrated via Simulation The four remaining parameters, $(\lambda_0, \lambda_1, \beta, \alpha)$, are determined by matching four data moments: (i) a government debt-to-income ratio of 61.8%, (ii) an average interest rate spread of 200 basis points, (iii) volatility of consumption with respect to the volatility of output equal to 1.1, and (iv) an average term premium of long-term Spanish bonds equal to 2%.

Parameters set by simulation.

Results: Analysis of Results Table 3 summarizes the calibration outcomes, comparing model-generated moments with empirical data. The calibration effectively aligns the model with observed data moments. The table is split into two sections: targeted moments, which the model is designed to match, and non-targeted moments, which serve as an additional validation of the model's performance.

For targeted moments, the benchmark model closely approximates the data. The debt-to-GDP ratio (63% vs. 60% in data), mean spread (2.1% vs. 2.0%), consumption volatility relative to output (1.09 vs. 1.10), and average term premia (2.58% vs. 2.5%) all show strong alignment. This indicates the model's success in replicating key economic indicators for Spain.

The model also performs well for non-targeted moments. Consistent with features of small open economies, it captures consumption volatility exceeding output volatility, a characteristic observed in Spain, though less pronounced in advanced economies. The model replicates the counter-cyclical trade balance observed in the data, with a correlation of $\rho(nx/y, y) = -0.013$ compared to -0.075 in the data. Similarly, it produces counter-cyclical government interest rates relative to output ($\rho(r, y) = -0.190$ vs. -0.168), consistent with findings in Neumeyer and Perri (2005) and Uribe and Yue (2006). While the magnitudes of these correlations are not identical to the data, the alignment in direction is notable

Table 2: **Calibrated Parameter Values**

Description	Parameter	Value	Target / Source
Parameters selected directly			
Risk-free interest rate	r^*	0.04	US Treasury 10-Y
Coef. of Relative Risk Aversion	γ	2	Literature
Std. deviation of innovations	σ_ε	1.04%	Estimated
Income autocorrelation coef.	ρ	0.97	Estimated
Probability of Re-entry	ξ	0.083	Richmond and Dias (2009)
Duration of long-term bond	λ	0.0225	TO ADD
Parameters set by matching moments			
Discount factor	β		$\sigma(c)\sigma(y) = 1.1$
Income loss in default	ϕ_0		Debt/gdp = 60%
Income loss in default	ϕ_1		Int. rate spread = 2%
Term premia	α		Term premia = 2.5%

Notes: Appendix **??** *provides further details on the calibration strategy and data sources.*

given their non-targeted nature. The model further generates a low default rate (0.258%), providing a realistic representation of sovereign risk dynamics.

Overall, Table 3 demonstrates that the benchmark model effectively captures both the long-run and business cycle characteristics of a small open economy with sovereign risk, closely matching both the sign and magnitude of key empirical moments.

5 Treasury Debt Composition and Central Bank Holdings

This section presents the primary findings from our calibrated model. We analyze the sovereign Treasury's optimal debt maturity structure in response to varying levels of Central Bank long-term bond holdings (b_{cb}), a proxy for policies such as Quantitative Easing (QE) or Tightening (QT). The analysis focuses on elucidating the economic mechanisms that drive the Treasury's decisions and comparing the model's predictions with key empirical regularities.

5.1 Optimal Debt Maturity Structure and Central Bank Holdings

A central empirical observation motivating this study is that the share of short-term public debt tends to *decrease* (implying an increase in the long-term debt share) during periods of significant central bank asset purchases. Our ATI model aims to provide a theoretical framework for understanding such strategic debt management by the Treasury.

Figure (1) demonstrates the model's core prediction regarding debt composition. It plots the average long-term debt fraction chosen by the Treasury (of total newly issued debt, conditional on active issuance) across different prevailing states of central bank holdings, b_{cb}. These b_{cb} states are grouped

Table 3: **Long-run Effects of introducing Non-enforceable Fiscal Rules**

Description	Data	Benchmark
Targeted Moments		
Debt-to-GDP (%)	60	63
Mean Spread (%)	2	2.1
$\sigma(c)/\sigma(y)$	1.10	1.09
avg. Term Premia	2.5	2.58
Non-targeted Moments		
$\sigma(nx/y)/\sigma(y)$	0.383	0.415
$\rho(c,y)$	0.97	0.924
$\rho(nx/y,y)$	−0.075	−0.013
$\rho(r,y)$	−0.168	−0.190
$\rho(ST/Total, B_{cb})$	−0.04	−0.123
avg. default rate		0.258%

Notes: Data on household consumption (c), GDP (y), the trade balance (nx) and interest rates on government debt (r) is taken from the Eurostat database. The data spans the period between 2000:Q1 and 2023:Q4. Consumption and GDP are expressed in logarithms. The logarithm of household consumption and the logarithm of GDP are de-trended using a Hodrick-Prescott filter. $\sigma(x)$ denotes the standard deviation of variable x and $\rho(x,y)$ denotes the correlation between variable x and variable y. The table reports model-based statistics using 100.000 *simulations in which the first* 5.000 *simulations are discarded.*

from the lowest observed levels (Q1) to the highest (Q5) in our simulations.

As illustrated, the model predicts a clear positive relationship: as central bank holdings b_{cb} increase (moving from Q1 to Q5), the Treasury optimally increases the fraction of long-term debt in its new issuance. Numerically, the average long-term debt fraction rises from approximately 31.8% when b_{cb} is in its lowest state (Q1) to about 40.6% when b_{cb} is in its highest state (Q5). Correspondingly, the short-term debt fraction decreases with higher b_{cb}. This is further supported by a simulated contemporaneous correlation of approximately -0.12 between the short-term debt fraction ($b_s/(b_s + b_l)$ of new issues) and the level of b_{cb}.

5.2 Mechanisms Driving the Treasury's Response

The Treasury's decision to alter its debt maturity structure in response to Central Bank holdings is not a trivial reaction to a single price signal but emerges from a complex dynamic optimization. Several interacting economic mechanisms contribute to this outcome.

A primary driver is the *term premium channel.* In our model, the term premium on long-term debt,

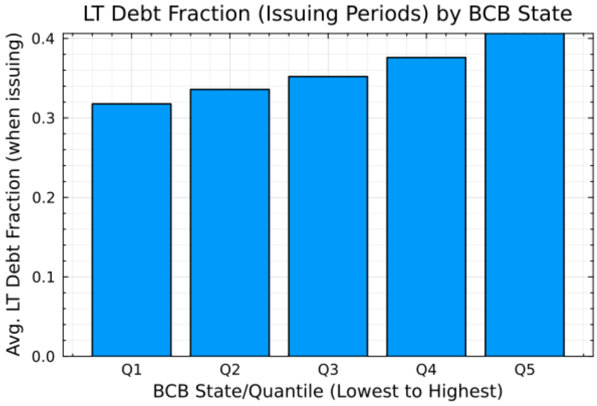

Figure 1: Average Long-Term Debt Fraction (of total new issuance) Conditional on Central Bank Holdings (b_{cb}) State. BCB States are ordered from lowest (Q1) to highest (Q5) holdings.

given by (**??**), is directly influenced by central bank actions. An increase in central bank holdings of long term debt, b_{cb}, characteristic of QE, reduces the net supply of long-term bonds available to the private market. This reduction, via the portfolio balance effect captured by (**??**), tends to lower the term premium. A lower term premium, in turn, decreases a component of the discount factor applied to future cash flows of long-term bonds, thereby increasing their issuance price ($q_{l,t}$) and lowering their effective yield. Our results (Appendix 9.1 figure 8) confirm that q_l indeed becomes more favorable as b_{cb} increases, providing a direct incentive for the Treasury to tilt its issuance towards longer maturities. The model demonstrates that this channel is operative and quantitatively relevant.

However, this incentive operates within an environment where the Treasury also considers *sovereign default risk*. The price of any debt instrument, whether short-term (q_s) or long-term (q_l), incorporates the market's expectation of the sovereign's future default probability. While an increase in longer-term central bank bond holdings tends to lower the current term premium (or set it to zero if $b_{l,t} \leq b_{cb,t}$), thereby exerting upward pressure on long-term bond prices via the discount factor, this effect can be counteracted or even reversed if higher central bank holdings also leads to a substantial increase in the market's expectation of future default. A higher perceived default probability would directly reduce the expected payoff component of both q_s and q_l. Our current model calibration, however, results in generally low average default rates across all b_{cb} states, as depicted in Figure 2. The average default rate varies moderately, from approximately 0.10% to 0.43% across the range of b_{cb} states, indicating that pervasive default risk does not negate the influence of the term premium channel in this specific calibration.

Indeed, in specific adverse regions of the state space (e.g., characterized by very low income and high initial debt, we observed scenarios where higher levels of central bank holdings were associated with a significantly increased probability of future default. In such instances, the negative impact of heightened default expectations on bond prices overwhelmed the beneficial effect of a reduced term premium. This resulted in both q_s and q_l *decreasing* as b_{cb} increased, as illustrated conceptually in Figure (9) in the appendix. The mechanism for such an outcome is a *self-reinforcing loop*: if lenders anticipate that high current central bank bond holdings (which is a persistent process) implies a higher likelihood of future period default when the sovereign attempts to roll over or issue new debt in a future state of the world, investors will offer poor terms (low q_s and q_l) today. These poor terms can, in turn, make default the optimal choice for the sovereign in the current state.

The Treasury also constantly evaluates *short-term debt costs* and associated *rollover risks*. Short-term debt typically does not carry a term premium and may offer lower baseline yields. Nonetheless, it exposes the sovereign to the risk of having to refinance frequently, possibly under adverse future market conditions (i.e., facing low future short-term bond prices). The decision to increase the long-term share when b_{cb} is high implies that, at the treasuries optimal decision, the benefit from cheaper long-term financing (via lower term-premium) outweighs the appeal of short-term debt, even if short-term debt remains accessible at relatively favorable terms (evidenced by generally high q_s values in many states). Our model captures the cost of short-term debt via $q_s \propto \mathbb{E}_t[1 - d(S')]$, thus linking it to default expectations rather than an explicit liquidity premium tied to b_{cb}.

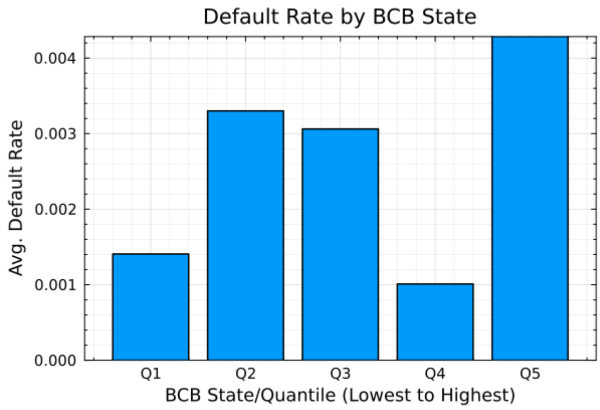

Figure 2: Average Default Rate Conditional on Central Bank Holdings (b_{cb}) State. BCB States are ordered from lowest (Q1) to highest (Q5) holdings.

5.3 Model Alignment with the empirical results

The model's prediction that higher central bank bond holdings (b_{cb}, akin to QE) lead to an increase in the long-term debt fraction (decrease in short-term debt fraction) is qualitatively consistent with our empirical finding for Spain. This suggests that the term premium/portfolio balance channel, whereby central bank purchases make long-term debt relatively more attractive to the sovereign issuer, is a potent force in explaining observed debt management patterns.

The simulated correlation between short-term debt fraction and b_{cb} is relatively modest (-0.12). This indicates that while the term premium channel operates in the theoretically expected direction and aligns with the data, other factors within the model (such as income shocks, existing debt levels, and the precise state-contingent costs of short-term vs. long-term debt derived from default risk) also significantly influence the Treasury's optimal debt composition. The Treasury is not solely reacting to the term premium induced by b_{cb} but is solving a complex dynamic optimization problem.

The results highlight the importance of the interplay between central bank actions and the sovereign's expectations regarding future financing conditions and default risk. In calibrations where high b_{cb} was perceived by the model's agents as leading to very high future default risk, the term premium channel was overwhelmed, and market access was shut down. The current calibration, which results in low overall default rates, allows the term premium effect to manifest more clearly, aligning the model with empirical observations on debt maturity.

6 Isolating the Term Premium Channel: A Counterfactual Analysis

The *benchmark model* demonstrates that an activist Treasury shifts its debt maturity structure towards longer terms when Central Bank holdings (b_{cb}) are higher, a response attributed to the b_{cb}-induced variations in the endogenous term premium given by (**??**). To more rigorously isolate and quantify the precise role of this *variable* term premium channel, we conduct a counterfactual analysis. We compare the benchmark model against an alternative specification where the term premium is *exogenously fixed* at a constant, positive level, $\bar{\zeta}$. For this experiment, $\bar{\zeta}$ is set to 0.025 (or 2.5% per period), a value that aligns with the average empirical term premium targeted in our overall model calibration (see Table **??**). In the Model with *Fixed Term-premium*, while long-term debt consistently carries this additional cost $\bar{\zeta}$ relative to short-term debt (beyond default risk and r^*), this premium does *not* respond to changes in the level of central bank's bond holdings or the Treasury's outstanding long-term debt. All other parameters of the model with *Fixed Term-premium* are identical to the benchmark Model. The differences in outcomes between both models can thus be directly attributed to the influence of the central bank unconventional monetary policies on the term premium and asset prices.

6.1 Impact on Long-Term Bond Prices (q_l)

Figure (4) presents the price of newly issued long-term debt (q_l) as a function of the current level of central bank holdings, for a representative level of current income (y_t) and chosen future debt levels ($b'_{s,t+1}, b'_{l,t+1}$). In the model with fixed term-premium (dashed line), q_l is constant across all levels of central bank holdings at approximately 0.197. This stability is expected, as the fixed $\bar{\zeta} = 0.025$ and the expected stream of default risk (which, for this slice, appears largely insensitive to current level of central bank holdings once ζ is fixed) are the primary determinants beyond r^*.

In contrast, for the benchmark Model (solid line), q_l exhibits significant sensitivity to the level of central bank holdings. When the level of central bank holdings is low (e.g., $b_{cb,t} \approx 0.01$), the endogenous term premium is at its highest for a given level of long-term bonds, resulting in a substantially lower long-term bond prices (around 0.15) compared to the alternative model. As level of central bank holdings increases, the term-premium decreases, causing the long-term bond prices in Model A to rise, eventually surpassing q_l from Model C when level of central bank holdings is high (e.g., $q_l^A \approx 0.28$ at $b_{cb,t} \approx 0.5$, whereas $q_l^B \approx 0.197$).

The price difference, $q_l^{Bench} - q_l^{\text{Fix TP}}$, plotted in Figure (3), directly quantifies this impact. It is positive when low levels of central bank holdings makes Model A's term premium, $\zeta_t > \bar{\zeta}$, and negative when high central bank holdings makes Model A's term-premium, $\zeta_t < \bar{\zeta}$ (often $\zeta_t = 0$).

On average across simulations Table (4), the mean realized price of long-term bonds in the Benchmark model A is 0.227, while in the fixed term-premium model it is 0.197. This suggests that the Treasury in the benchmark model, by actively responding to the variable term-premium, achieves, on average, better financing terms for its long-term debt than if it always faced the constant $\bar{\zeta} = 0.025$. This is because the average realized term-premium in the benchmark economy's simulations is approximately 1.74% (per period), which is lower than the imposed 2.5% in the economy with a fix term-premium. The Treasury in the benchmark economy effectively times its long-term issuance to periods where its endogenous ζ_t is relatively low.

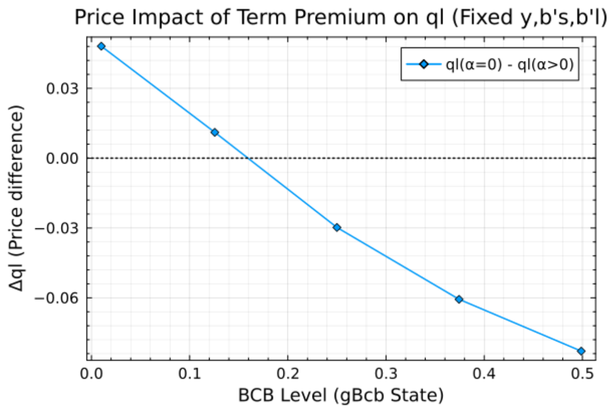

Figure 3: q_l (Fixed y, b_s', b_l'): Benchmark vs. Fixed Term Premium Model.

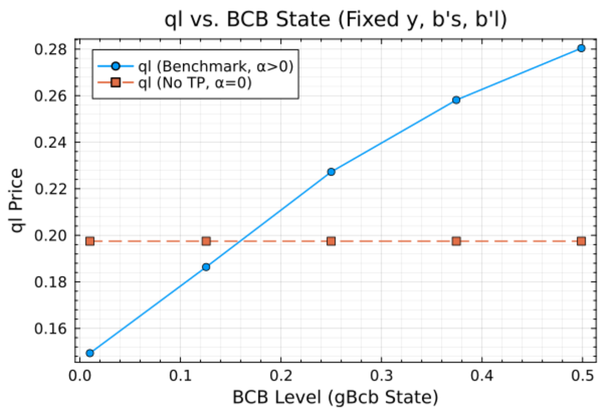

Figure 4: q_l vs. b_{cb} State (Fixed y, b_s', b_l'): Benchmark vs. Fixed Term Premium Model.

Table 4: **Long-run comparison between the benchmark and the economy with fixed term-premium**

Description	Data	Benchmark	Fixed Term-Premim
Targeted Moments			
Debt-to-GDP (%)	60	63	64.7
Mean Spread (%)	2	2.1	2.1
$\sigma(c)/\sigma(y)$	1.10	1.09	1.09
avg. Term Premia	2.5	2.58	0.0
Non-targeted Moments			
$\sigma(nx/y)/\sigma(y)$	0.383	0.415	0.41
$\rho(c, y)$	0.97	0.924	0.925
$\rho(nx/y, y)$	-0.075	-0.013	-0.014
$\rho(r, y)$	-0.168	-0.190	-0.193
$\rho(ST/Total, B_{cb})$	-0.04	-0.123	-0.003
avg. default rate		0.258%	0.171%

Notes: Data on household consumption (c), GDP (y), the trade balance (nx) and interest rates on government debt (r) is taken from the Eurostat database. The data spans the period between 2000:Q1 and 2023:Q4. Consumption and GDP are expressed in logarithms. The logarithm of household consumption and the logarithm of GDP are detrended using a Hodrick-Prescott filter. $\sigma(x)$ denotes the standard deviation of variable x and $\rho(x, y)$ denotes the correlation between variable x and variable y. The table reports model-based statistics using 100.000 *simulations in which the first* 5.000 *simulations are discarded.*

6.2 Impact on Treasury Debt Maturity Structure

The critical test is how these differing price dynamics translate into the Treasury's optimal debt maturity choices. Figure (5) compares the average long-term debt fraction (conditional on issuance) across the different central bank bond holdings for both economies.

As previously established in Figure (1), in the benchmark economy, the long-term debt fraction systematically increases with central bank bond holdings, driven by the more favorable long-term bond prices when these central bank holdings are high. In stark contrast, Figure (5) shows that when the term premium is constant and thus invariant to central bank actions on the bond market, the Treasury's chosen long-term debt fraction is essentially flat across all central bank levels of bond holdings, averaging around 35.9%. The modest correlation of the ratio of short-term bonds with central-bank bond holdings is approximately -0.12 in the benchmark economy and reduces to virtually zero (≈ -0.003) in the economy with fixed term-premium (Table 4).

This comparison powerfully illustrates the central role of the b_{cb}-sensitive term premium. While both models incorporate a cost for issuing long-term debt via a term premium, only in the benchmark economy the Treasury as an incentive and mechanism to actively adjust its maturity structure in direct response to central bank actions that alter this premium. The Treasury in the benchmark economy

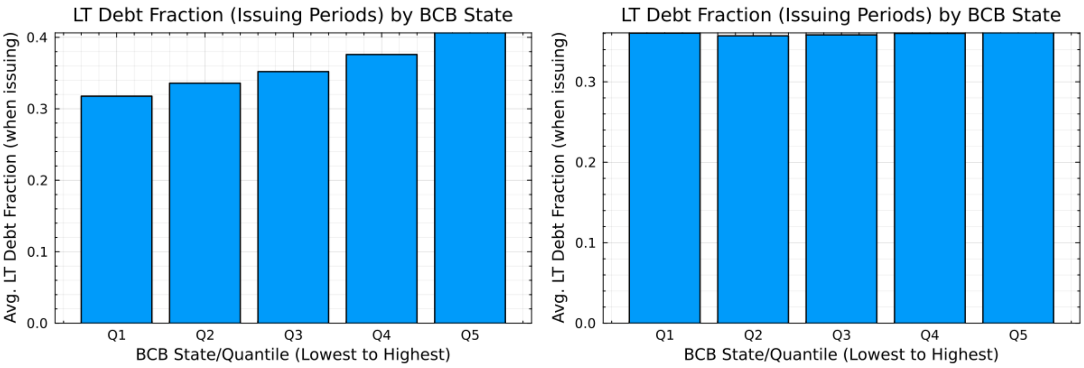

Figure 5: Average long-term Debt Fraction vs. b_{cb} State: Comparison between Benchmark Model with Endogenous Term Premium (a) and Fixed Term Premium Model (b).

lengthens its debt maturity when b_{cb} is high precisely because doing so becomes relatively cheaper through the reduction in the term premium. This strategic response is absent in the economy with a fixed term-premium where b_{cb} provides no such state-contingent pricing advantage for long-term debt.

6.3 Overall Debt Levels and Default Risk

Table (4) reveals that the average debt-to-GDP ratio is slightly lower in the benchmark economy (63.0%) compared to the economy with a fixed term-premium (64.7%). This suggests that the variability of the term premium, and the associated uncertainty or need for active management, might lead to slightly more conservative overall debt postures. Interestingly, the alternative economy exhibits the lowest average default rate (0.171%) compared to Model A (0.258%) and a model with no term premium at all (0.330%, from Appendix Z.Z). This could imply that a stable, predictable (even if positive) cost structure for long-term debt might contribute to more stable fiscal outcomes in terms of default, even if it removes some opportunities for opportunistic maturity timing. This particular finding warrants further investigation but highlights that the implications of different term premium structures extend beyond just the debt maturity mix.

6.4 The Primacy of the Variable Term Premium Channel

The counterfactual analysis strongly supports the hypothesis that the empirically observed tendency for sovereigns to increase the maturity of their debt during periods of QE is driven by the effect of central bank asset purchases on the term premium. Our benchmark economy, featuring an endogenous term premium responsive to net long-term debt supply (and thus to b_{cb}), successfully generates this qualitative pattern. When this specific channel of influence is neutralized by fixing the term premium , the systematic relationship between central bank bond holdings and debt maturity vanishes, even if an average term premium cost for long-term debt persists.

This underscores that it is not merely the existence of a term premium, but its sensitivity to central bank operations, that provides the key incentive for an activist Treasury to alter its issuance strategy in response to policies like QE or QT. The Treasury's ability to *time the market* for long-term debt, issuing more when the b_{cb}-induced component of the term premium is low, is central to this dynamic.

7 The Economic Value of Strategic vs. Rule-Bound Debt Management

Our benchmark economy demonstrates that a sovereign optimizing its debt level and maturity structure can strategically respond to central bank policies, particularly by adjusting its issuance mix to capitalize on variations in the term premium induced by central bank asset holdings. To quantify the economic implications of this flexibility, we compare the *benchmark economy* outcomes with those from a counterfactual *rules-bound economy*, highlighting differences in debt levels, default risk, consumption dynamics, and responsiveness to monetary policy shocks.

7.1 Defining the Passive Issuance Policy

Traditionally, treasuries follow a passive debt issuance path where sovereign debt managers follow a fixed rule for the debt maturity mix. We label this the rule-bound economy. In the *rule-bound economy*, the sovereign Treasury, while still making optimal decisions regarding the total level of new borrowing and whether to default, is constrained to issue new debt in a fixed maturity mix. Specifically, any new total borrowing required, B_{t+1}, is allocated such that a constant fraction $\bar{\theta}$ is financed through short-term (ST) bills and the remaining $(1-\bar{\theta})$ through long-term (LT) bonds:

$$b_{s,t+1} = \bar{\theta} \cdot B_{t+1}, \quad b_{l,t+1} = (1-\bar{\theta}) \cdot B_{t+1}.$$

For this analysis, $\bar{\theta}$ is calibrated to 0.3, reflecting the average short-term debt share in new issuance observed in our empirical data for Spain. Bond prices (q_s, q_l) remain endogenously determined by market expectations of default under this passive policy regime, and the term premium still affects the price of the long-term debt portion. The key difference is that the Treasury cannot deviate from the $\bar{\theta}$ rule, regardless of prevailing market conditions or the level of central bank bond holdings.

7.2 Macroeconomic Stability, Default Risk, and Debt Levels

Table (5) presents key long-run simulated moments for the *benchmark model* and the *passive model*, alongside relevant empirical data and other counterfactuals discussed previously.

A stark contrast emerges between the two economies. The *rule-bound economy* results in an average debt-to-GDP ratio of 66.8%, higher than that of the economy with a strategic debt managing office (63.0%) and the empirical target (60%). This high debt accumulation stems from the rule-bound economy's tendency to borrow the maximum allowable amount in all income states, as discussed below. Despite this substantial debt burden, the rule-bound economy achieves a simulated average default rate of zero. This outcome is sustained because, as illustrated in Figure (6), the short-term spread faced by the passive government is effectively zero across all income states. Lenders perceive negligible default risk on its short-term obligations, allowing continuous debt rollover, albeit with significant economic costs.

The *rule-bound economy* exhibits lower consumption volatility relative to income, with $\sigma(c)/\sigma(y) = 1.005$, compared to $\sigma(c)/\sigma(y) = 1.09$ in the benchmark model. However, the rule-bound economy's correlation between consumption and income, $\rho(c, y) = 1.0$, reveals that consumption perfectly tracks income fluctuations, indicating a lack of active smoothing. In contrast, the economy with the strategic

Table 5: **Simulated Long-Run Moments: Active Issuance vs. Passive Issuance**

Description	Data	Benchmark ($\alpha > 0$)	Rule-bound Issuance ($\bar{\theta} = 0.3$)
Debt-to-GDP (%)	60	63.0	66.8
$\sigma(c)/\sigma(y)$	1.10	1.09	1.005
Avg. Term Premia (bps)[d]	250	258	499
$\rho(c, y)$	0.97	0.924	1.000
$\sigma(nx/y)/\sigma(y)$	0.383	0.415	0.011
$\rho(nx/y, y)$	−0.075	−0.013	−0.43
Avg. Default Rate (%)	n/a	0.258	0.000

Notes: Model statistics from 100,000 simulations post burn-in. See main text/Appendix for data sources and full definitions. [a]ST Spread derived from model's q_s. [b]Empirical ST spread target (e.g., 2%). [c]Model-implied ST spread for Passive is effectively zero. [d]Realized $\zeta_t = \alpha \max(0, b_{l,t} - b_{cb,t})$. [e]ST Rate derived from model's q_s and r^*. [f]Correlation with constant ST rate is zero. [g]ST Fraction of new issues is fixed by rule in Passive model.

debt manager has lower $\rho(c, y) = 0.924$ suggesting that consumption is less tied to income shocks, reflecting countercyclical borrowing that decouples consumption from income, albeit at the cost of higher relative volatility due to debt repayment dynamics and default risk (0.258%).

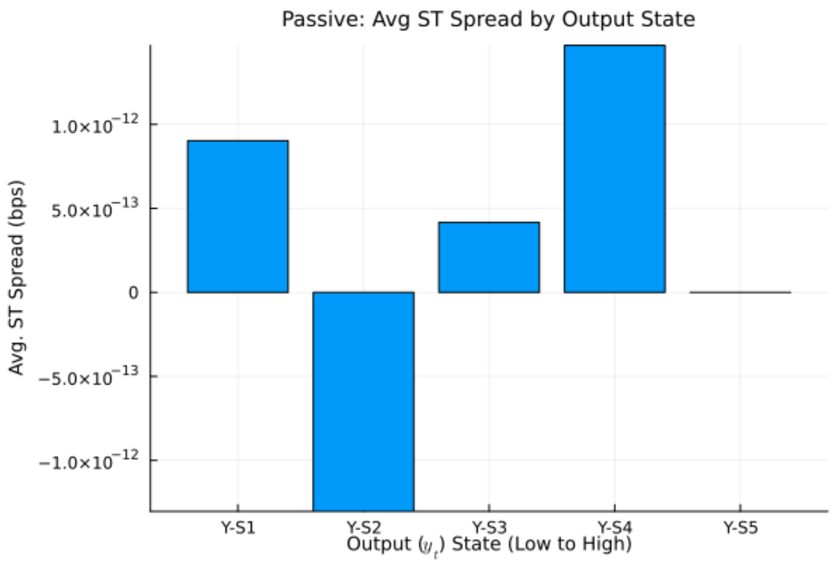

Figure 6: Passive Model: Average Short-Term Bill Spread by Output State.

7.3 Debt Issuance Strategies and Response to Shocks

The divergent macroeconomic outcomes are rooted in fundamentally different debt issuance strategies, particularly in response to income fluctuations. Figure (7) illustrates the average total new debt chosen (B'_{total}) by each government across different income states.

The benchmark economy exhibits a procyclical borrowing pattern, increasing borrowing as income rises from low to high income states. This behavior, typical of developing markets, where credit access

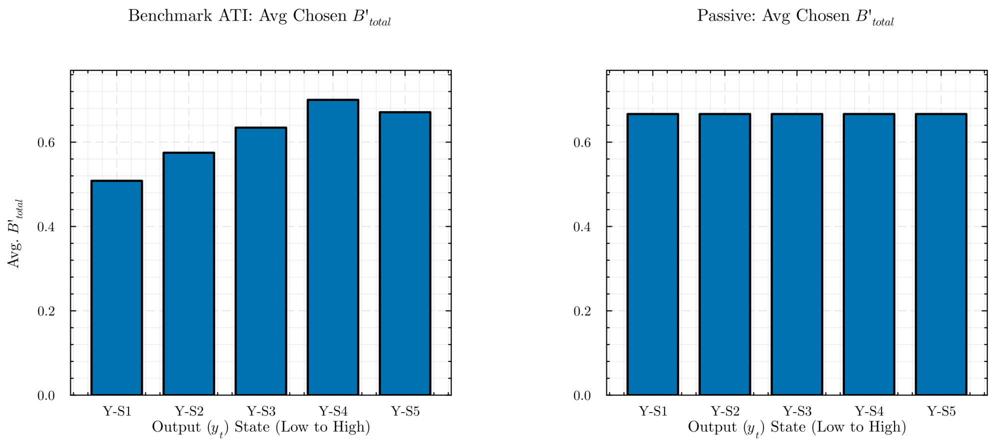

Figure 7: Average Total New Debt (B'_{total}) Chosen vs. Output (y_t) State.

tightens during downturns due to heightened risk premia, as noted in Neumeyer and Perri (2005). In low-income states, higher default probabilities reduce short-term and long-term bond prices, limiting borrowing capacity and forcing consumption to adjust closely to income. In high-income states, lower default risk and higher bond prices enable greater borrowing, increasing total debt issuance. This procyclical pattern contributes to higher consumption volatility ($\sigma(c)/\sigma(y) = 1.09$) relative to income, as consumption falls significantly in low-income states but may rise in high-income states due to increased borrowing. However, the correlation $\rho(c, y) = 0.924$ indicates some decoupling of consumption from income shocks, reflecting the government's ability to adjust debt maturities (e.g., increasing long-term debt during QE) to partially mitigate income fluctuations.

In contrast, the rule-bound economy, with its fixed maturity mix, chooses the maximum allowable new debt ($B'_{total} = 0.6$, or 60% of average GDP) in every income state, as shown in Figure (7). This constant borrowing, effectively procyclical in its lack of state-contingent adjustment, explains the high debt-to-GDP ratio (66.8%) and low consumption volatility ($\sigma(c)/\sigma(y) = 1.005$), as consumption tracks income closely ($\rho(c, y) = 1.0$). The zero default risk ensures high short-term bond prices ($q_s \approx 1/(1 + r)$), allowing continuous debt rollover, which stabilizes consumption relative to income but prevents active smoothing.

The rule-bound economy's maximal borrowing, even from a low-debt position (Figure 7), stems from the fixed issuance mix's interaction with the government's valuation of future states. With unconstrained borrowing and risk-free short-term financing, the utility gains from high consumption today outweigh discounted future costs, as default is avoided due to high implicit default costs in our calibration. The fixed mix reduces the appeal of state-contingent borrowing choices compared to the benchmark economy, where maturity optimization allows responsiveness to income and central bank policies.

7.4 The Value of Strategic Debt Management

The comparison between the rules-bound and strategic debt management paints a unclear picture regarding the superior debt management strategy and tells a **story of divergence** through the interplay of **strategic freedom and default risk versus rigid optimization and risk-free stability**.

The Strategist, constrained by a positive default probability, adopts a procyclical borrowing strategy, amplifying consumption volatility due to limited borrowing in low-income states and surges in high-income states. However, its ability to adjust debt maturities – exploiting QE to increase long-term debt issuance – delivers partial consumption smoothing, mitigating the impact of income shocks on household welfare.

The overall picture shows a rules-bound economy without the ability to smooth consumption resembling a hand-to-mouth economy although it does have access to international financial markets. There are two reasons for this inability to smooth consumption. First, the rules-bound economy has higher a term-premium. Second, by design, the economy has less flexibility to respond to monetary policy decisions. These two reasons lead to a more countercyclical trade balance.

The benchmark economy's can respond to central bank holdings, increasing long-term debt issuance during QE when the term premium falls. During Quantitative Easing (QE), when central bank purchases reduce the term premium, the Strategist shifts toward long-term debt, increasing its share from 31.8% to 40.6% of new issuance. This strategic response, evidenced by a negative correlation between the short-term debt fraction and central bank holdings ($\rho(\text{ST Frac new issues}, b_{cb}) = -0.123$), secures higher bond prices (q_l), boosting borrowing proceeds and enabling **partial consumption smoothing** ($\rho(c, y) = 0.924$). For risk-averse households with a coefficient of relative risk aversion $\gamma = 2$, this decoupling of consumption from income shocks mitigates welfare losses, though volatility remains elevated due to procyclical borrowing constraints. The **term premium** (258 bps) is moderate, reflecting the Strategist's ability to align long-term debt issuance with favorable market conditions during QE, and the countercyclical short-term rate ($\rho(\text{ST Rate}, y) = -0.190$) underscores the influence of default risk on borrowing costs.

Finally, the benchmark model demonstrates resilience to QT shocks (Section (**??**)), reducing borrowing and shifting to short-term debt to mitigate consumption declines, while the passive model suffers persistent consumption drops due to its inflexibility.

The Rule-Bound Treasurer operates under a rigid framework, issuing debt with a **fixed maturity mix** and optimizing total borrowing without default risk. Lenders offer risk-free short-term debt ($q_s \approx 1/(1 + r)$), ensuring stable borrowing terms. Unlike the Strategist's procyclical approach, the Rule-Bound Treasurer adjusts borrowing to income states, likely reducing total borrowing in high-income states to maintain a moderate debt level, as the 66.8% ratio suggests optimization within the fixed mix. This adjustment, constrained by the fixed short-term and long-term debt shares, stabilizes consumption, yielding **low consumption volatility** ($\sigma(c)/\sigma(y) = 1.005$). However, **no active smoothing** occurs ($\rho(c, y) = 1.000$), as consumption tracks income perfectly, offering no buffer against shocks for households. The fixed mix prevents responsiveness to QE or Quantitative Tightening (QT), as evidenced by the zero correlation between the short-term debt fraction and central bank holdings ($\rho(\text{ST Frac new issues}, b_{cb}) = 0.000$). The **high term premium** (499 bps) reflects constant exposure to long-term debt, inflating financing costs compared to the Strategist's adaptive approach, as the fixed mix locks in a significant long-term debt share regardless of market conditions.

However, how can a passive treasurer that cannot adjust maturities and faces higher term-premium costs sustain higher debt can sustain higher debt levels at a zero default risk? The gains from these rules-based approach are based on, first, a fix and low fraction of short-term debt issuance, leading to lower roll-over risk and zero default risk. These lack of default risk raises asset prices. Second, the fix debt mix constrains the economy to a lower volatility of consumption relative to output. An aggregate economy that is less volatile also leads to a lower willingness to default which is able to sustain higher asset prices via lower default risk. Both reasons lead to an ability to borrow more and sustain higher consumption.

7.5 Welfare Evaluation

Evaluating **welfare**, the Strategist likely outperforms, as smoothing ($\rho(c, y) = 0.924$) mitigates shock impacts for $\gamma = 2$, despite volatility.

TO FINISH

8 Conclusion

This paper investigates how sovereign governments should strategically manage their debt maturity structure when central banks actively participate in bond markets. Through empirical analysis of Spanish debt issuance and a quantitative sovereign debt model, we provide compelling evidence for an *Activist Treasury*, a maturity management strategy whereby the government adjusts the composition of debt issuance in response to changes in central bank bond holdings. Motivated by the unprecedented expansion and subsequent unwinding of the ECB's balance sheet in the euro area, I analyze both empirically and theoretically how a sovereign treasury may react to fluctuations in the term premium driven by unconventional monetary policy.

The empirical evidence, based on monthly Spanish data from 2009 to 2024, shows that the maturity structure of issuance responds systematically to ECB asset purchases. When the ECB increases its holdings of Spanish sovereign bonds, the Spanish Treasury reduces its reliance on short-term debt, thereby lengthening the maturity profile of new issuance. This behavior is robust across specifications and persists through the transition to Quantitative Tightening. These findings are consistent with the hypothesis that the Treasury exploits the compression in the term premium induced by QE to lock in cheaper long-term financing.

To interpret and quantify the welfare implications of this behavior, I develop a sovereign default model with both short- and long-term debt, augmented by a portfolio balance channel. Central bank purchases reduce the long-term yield by decreasing the net supply of long-term bonds held by private investors. The government optimally adjusts issuance in response to these changes: it increases long-term borrowing during QE, when long-term rates are depressed, and shortens maturities during QT to avoid elevated term premia. Crucially, when we eliminate the variable term premium in a counterfactual analysis, this systematic relationship vanishes entirely, confirming that the portfolio balance channel drives the strategic behavior.

The welfare analysis reveals nuanced trade-offs between activist and passive debt management. The activist approach achieves meaningful consumption smoothing (correlation of 0.924 versus perfect correlation under passive policy) and benefits from lower average term premia through strategic market timing. However, this flexibility comes at the cost of higher default risk and lower sustainable debt levels. The passive approach, despite facing higher financing costs, sustains higher debt levels at zero default risk due to reduced rollover risk and lower aggregate volatility.

The policy implications are significant for high-debt economies, particularly in currency unions like the Eurozone. By lengthening debt maturities during QE, treasuries can lock in lower borrowing costs, reducing refinancing risks and saving an estimated 12 billion annually for Spain during PSPP. During QT, shifting to short-term debt mitigates fiscal strain from rising yields. These strategies enhance fiscal space and substitute for monetary autonomy, but require coordination with central banks to avoid conflicting objectives, such as premature QT exacerbating fiscal costs. Policymakers must also manage the trade-off between strategic flexibility and market predictability, as frequent maturity adjustments may increase yield volatility by 0.3%.

Our analysis, however, is not without limitations. The model abstracts from several potentially important factors, including political economy considerations that may influence debt management decisions, the role of inflation expectations, and the potential for a strategic game between the Treasury and the central bank. These simplifications provide a clear and tractable framework but also open up promising avenues for future research. One could extend the model to a game-theoretic setting to explicitly analyze the strategic interactions and potential conflicts between the two authorities. Another fruitful extension would be to incorporate a richer model of inflation dynamics to explore how the coordination of debt management and monetary policy affects price stability. Finally, examining these dynamics in a multi-country setting could shed light on the international spillovers of these increasingly important policy interactions.

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9 Appendix

9.1 Figures

10 Preferred–Habitat Dealers and Endogenous Bond Prices

This appendix derives the short–term liquidity premium $h_{S,t}$ and the long–term *term premium* τ_t from the optimal behaviour of international *preferred-habitat* dealers who intermediate the supply of sovereign debt. The resulting expressions slot directly into the pricing formulas (14)–(15) used in the main text.

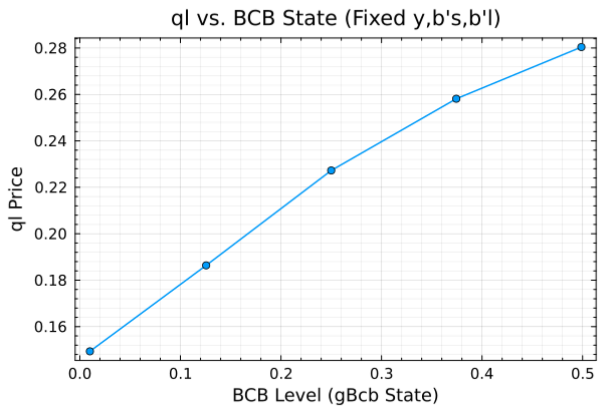

Figure 8

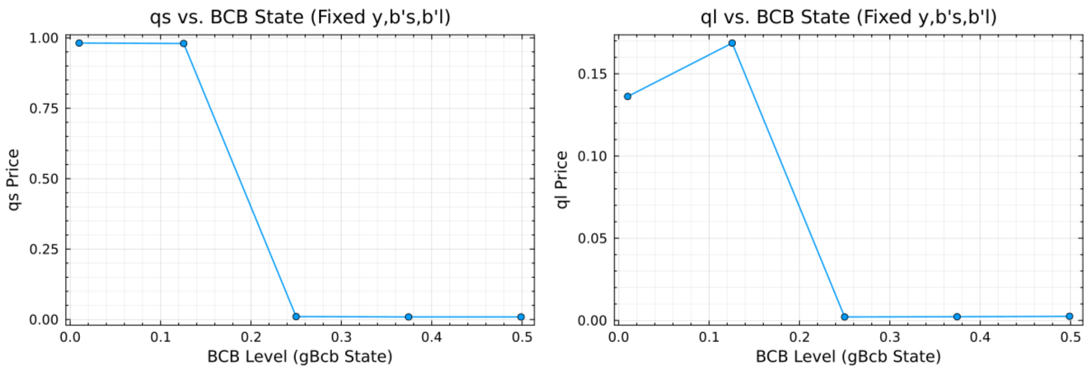

Figure 9

Two representative risk–neutral dealers trade a one-period bill and a consol (perpetuity). Their holdings at the end of t are b_t^S and b_t^L, respectively. Adjusting inventories incurs a quadratic cost

$$C_i\left(b_{t+1}^i - b_t^i\right) \;=\; \tfrac{1}{2}\,\kappa_i\left(b_{t+1}^i - b_t^i\right)^2, \qquad i \in \{S, L\},$$

where $\kappa_i > 0$ measures habitat *stickiness.*

Dealer optimisation At date t dealer i chooses b_{t+1}^i knowing today's price $q_{i,t}$ and tomorrow's stochastic gross payoff R_{t+1}^i:

$$\max_{b_{t+1}^i} E_t\left[R_{t+1}^i\, b_{t+1}^i\right] - q_{i,t}\, b_{t+1}^i - \tfrac{1}{2}\,\kappa_i\left(b_{t+1}^i - b_t^i\right)^2.$$

The first-order condition is

$$q_{i,t} \;=\; E_t\left[R_{t+1}^i\right] \;-\; \kappa_i\left(b_{t+1}^i - b_t^i\right). \tag{16}$$

Market clearing Dealers must absorb the *net* public supply that is not on the central-bank balance sheet:

$$b_{t+1}^S = b_{S,t+1} - B_{t+1}^{cb}, \qquad b_{t+1}^L = b_{L,t+1} - B_{t+1}^{cb}.$$

With b_t^i additive or of zero mean, we approximate $b_{t+1}^i - b_t^i \approx b_{t+1}^i$ and insert market-clearing directly into (16) to get

$$
\begin{aligned}
q_{S,t} &= E_t\left[R^S_{t+1}\right] - \kappa_i\left(b_{S,t+1} - B^{cb}_{t+1}\right) &(17)\\
q_{L,t} &= E_t\left[R^L_{t+1}\right] - \kappa_i\left(b_{S,t+1} - B^{cb}_{t+1}\right) &(18)
\end{aligned}
$$

No-arbitrage pricing and yields. Let r be the risk-free rate, $h_{S,t}, \tau_t, h_{L,t}$ be the liquidity and term premia. Then

$$
\begin{aligned}
q_{S,t}\left(1 + r + h_{S,t}\right) &= E_t[R^S_{t+1}] = 1 - \lambda_{t+1}, &(19)\\
q_{L,t}\left(1 + r + \tau_t + h_{L,t}\right) &= E_t[R^L_{t+1}] = (1 - \lambda_{t+1})\big[\kappa + E_t[q_{L,t+1}]\big]. &(20)
\end{aligned}
$$

Here $1+r+h_{S,t}$ is the *gross yield* on bills, and $1+r+\tau_t+h_{L,t}$ the gross yield on consols. These no-arbitrage conditions (NAC) tell that The bond's *payoff* R_{t+1} captures only *default risk* (and coupon receipts), not the full return investors earn. The bond's *yield* (LHS in (19)) incorporates *risk-free rate*, *term premium* (ζ_t), and *liquidity wedge* ($h_{i,t}$). All term-structure effects ($r, \tau_t, h_{i,t}$) enter *outside* the expectation, in the discount factor, and do not belong in the random payoff R_{t+1}.

Thus,

$$
q_{S,t}\left(1 + r + h_{S,t}\right) = 1 - \lambda_{t+1} \quad \Longrightarrow \quad q_{S,t} = \frac{1 - \lambda_{t+1}}{1 + r + h_{S,t}}. \tag{DF}
$$

and

$$
q_{L,t}\left(1 + r + h_{L,t} + \zeta_t\right) = (1 - \lambda_{t+1})[\kappa + E_t[q_{L,t+1}]] \quad \Longrightarrow \quad q_{S,t} = \frac{1 - \lambda_{t+1}[\kappa + E_t[q_{L,t+1}]]}{1 + r + h_{S,t}}. \tag{DF}
$$

To reconcile (FOC) with (DF), observe that

$$
(1 + r + h_{S,t})\Big[(1 - \lambda_{t+1}) - \kappa_S \Delta b_{S,t+1}\Big] = 1 - \lambda_{t+1} \quad \text{where } \Delta b_{S,t+1} = b_{S,t+1} - B^{cb}_{t+1}.
$$

divide by $1 - \lambda - \kappa\Delta b$

$$
(1 + r + h_{S,t}) = \frac{1 - \lambda_{t+1}}{\Big[(1 - \lambda_{t+1}) - \kappa_S \Delta b_{S,t+1}\Big]} = 1 + \frac{\kappa_S \Delta b_{S,t+1}}{\Big[(1 - \lambda_{t+1}) - \kappa_S \Delta b_{S,t+1}\Big]}
$$

thus

$$
r + h_{S,t} = \frac{\kappa_S \Delta b_{S,t+1}}{\Big[(1 - \lambda_{t+1}) - \kappa_S \Delta b_{S,t+1}\Big]}
$$

Typically $\lambda \approx 0$ and $\kappa_S \Delta b_{S,t+1} << 1$ thus we approximate as

$$
r + h_{S,t} = \kappa_S \Delta b_{S,t+1}
$$

Plugging this back into (DF) reproduces exactly

$$
q_{S,t} = \frac{1 - \lambda_{t+1}}{1 + r + h_{S,t}}.
$$

Thus the inventory-cost term $\kappa_S\,(b_S - B^{cb})$ appears endogenously as the liquidity wedge $h_{S,t}$ in the standard discount-factor pricing form.

The same exercise will deliver

$$q_{L,t} \;=\; \frac{1-\lambda_{t+1}}{1+r+h_{L,t}+\zeta_t}.$$